



MCG 3131

Machine Design Project Report

Gear Reducer

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Abstract

This project focuses on the design and analysis of a stand-alone gear reducer capable of transforming an input speed of 750 rpm to an output speed of 75 rpm, utilizing an input power of 3 HP. The primary aim was to develop a reliable and efficient gear reduction system that transitions power from a vertical to a horizontal orientation. One potential application for the designed gearbox is in conveyor belt systems, where precise speed control and power transmission are essential for material handling processes.

The project involved a detailed examination of the gear reducer's components, including gears, bearings, shafts, and lubricants. Comprehensive analyses were conducted, encompassing gear selection, stress calculations, shaft and bearing analysis, and key design. Attention was given to fits and tolerances, critical speed, and maintenance considerations. Additional focus was placed on casing design, seals, retaining rings, and lubrication to ensure optimal performance and longevity. The calculation of security factors demonstrated the robustness of the design, confirming its reliability under various operational conditions.

This report provides valuable insights into the selection and analysis processes involved in creating an efficient gear reduction system. The final design offers promising reliability across diverse operational conditions.

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Nomenclature

Table 1: Nomenclature

Symbol	Definition
T_i	Torque
t_i	Number of teeth
N	Rotational speed
p	Pitch
m	Module
d_i	Diameter
F_i	Force
V	Pitch velocity
n	Angular speed
F_e	Equivalent force
F_r	Radial load
F_t	Thrust load
K_r	Life adjustment reliability
K_a	Application factor
L_R	Life corresponding to rated capacity
L	Life corresponding to radial load
C_{req}	Rated capacity

Glossary

Table 2: Glossary

Term	Definition
HP	Horsepower
Cs	Countershaft
RPM	Rotation per minute
AMGA	American Gear Manufacturers Association
SF	Safety Factor
lbf	Pounds-force
mm	Millimeter

Introduction

A stand-alone gear reducer refers to a mechanical device used to reduce the speed of an input shaft by transferring torque through a system of gears. In this project we would like to design a stand-alone gear reducer, vertical to horizontal, with an input power of 3 HP, an input speed of 750 rpm and an output speed of 75 rpm. Our stand-alone gear reducer must also be able to transmit power efficiently and be reliable in all kinds of circumstances.

In this project, we're going to talk about the various components of our gear reducer, such as gears, bearings, shafts, lubricants, and materials to adequately reduce the speed and balance the forces within the gear system; to help you understand our design, we'll be using SolidWorks to help you understand certain details. We're going to make the best possible product, considering various parameters such as cost, choice of material and different properties.

First, we'll look at gears, shafts, bearings, slopes and deflections. We'll carry out a complete analysis of these components, using calculations such as the number of teeth on the gears, the diameter of the various components, material properties, costs, etc. Then we'll talk about key design, tolerances and fits, and critical speed. Finally, we'll talk about the other considerations to be considered for our product.

1. Concept analysis

a) Theory basics

Gearboxes play a crucial role in motion and power control within mechanical systems, especially when there is a need to adjust the torque and speed transmitted between components. Their primary function is to efficiently and reliably transmit power from one part of a system to another. The design process involves selecting appropriate gear types, gear ratios, bearing arrangements, shaft dimensions, and lubrication methods tailored to the specific application.

For instance, bevel gears are used to change the direction of shaft rotation, while helical gears are favored for their smooth operation and higher contact ratios. Each component of the gearbox must be meticulously designed to ensure long-term efficiency and durability. Standards set by the American Gear Manufacturers Association (AGMA) are frequently consulted to determine the dimensions, tolerances, and stresses that gears must withstand.

In summary, designing a gearbox is a complex process that demands a thorough understanding of mechanical principles and careful consideration of the specific requirements of the application.

b) Project Requirements

Our design requires a vertical input and a horizontal output, necessitating the crossing of the shaft axes. To achieve this, we must incorporate bevel gears. This is the first crucial aspect of our design. The second consideration is the required gear ratio of 10:1, which means transforming an input speed of 750 RPM to an output speed of 75 RPM.

Given this high ratio, it is impractical to achieve it with a single set of gears. Therefore, we need a two-stage gearbox. The first stage will have a 5:1 ratio, and the second stage will have a 2:1 ratio. This combination allows us to achieve the overall 10:1 reduction.

With the types and alignment of gears determined, we need to calculate all aspects related to the gearbox to ensure its integrity and longevity. This includes determining the appropriate gear dimensions, bearing arrangements, shaft dimensions, and lubrication methods. Ensuring compliance with standards such as those set by the American Gear Manufacturers Association (AGMA) will help in achieving a reliable and durable gearbox design.

2. Design Analysis

Design Assumption

To guide the mathematical analysis and material selection for our system, below is a list of design assumptions. Some of these are based on examples provided in lectures and the class textbook.

- Highest manufacturing accuracy
- A precise gear ratio is not required
- All gears will be made from the same material
- For the Overload Correction Factor K_0 , the driven machinery is designed for moderate shock from a light shock power source
- We aim for 99% reliability in all materials
- Keep shafts as short as possible, with bearings close to the applied loads to reduce deflections and bending moments and increase critical speeds

- Place necessary stress raisers away from highly stressed shaft regions if possible. If not possible, use generous radii and good surface finishes. Consider local surface-strengthening processes such as shot peening or cold rolling
- Use inexpensive steels for deflection-critical shafts, as all steels have essentially the same elastic modulus
- We need to calculate every possible failure mode for the shaft, including bending, shear, fatigue, deflection, and torsion
- Shaft material will be AISI 4140
- We do not account for the mass of the gears
- The bearing must be able to operate at high speeds
- Ball bearings will be used to simplify analysis
- Since the application of the gearbox was not specified, we assume the design must operate under light to moderate shocks
- The mounting of the bearings (likely press-fit) will not affect their life
- Only one bearing on each shaft will carry an axial load.
- Our gear box will have a life expectancy of 6 year

3. Gear Analysis

Main gears disposition

The initial step in developing the design was to determine the speed reduction ratio. For this project, we were provided with specific parameters. Primarily, we needed to achieve vertical input and horizontal output. Additionally, we were given the following specifications:

- Input Power (P): $3 \text{ Hp} = 3 \times 745.7 = \mathbf{2237.1 \text{ Watt}}$
- Input velocity (ω_{in}): **750 rpm**
- Output velocity (ω_{out}): **75 rpm**

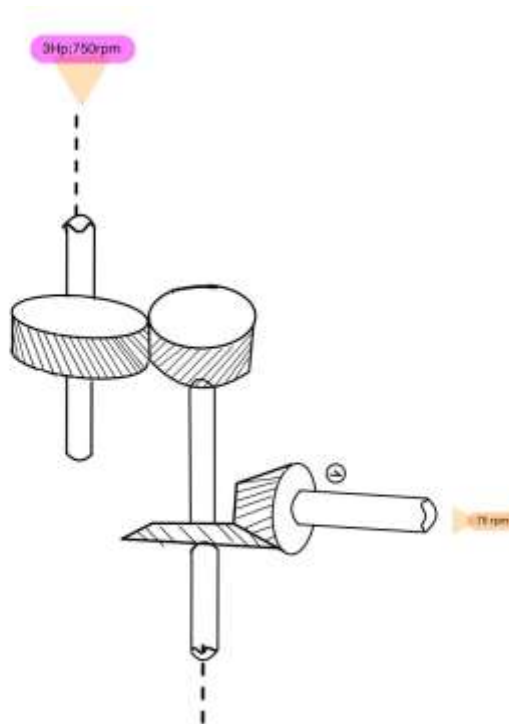


Figure 1: Representation of the speed reducer configuration

Based on the catalog, we need to determine the service factor by referencing a table that correlates the service factor with the operating conditions. If the gear reducer operates under moderate shock conditions, uniformly, and for no more than 8 hours per day, we can deduce from the table that the appropriate service factor is 1.0.

Service Factor	Operating Conditions
.8	Uniform — not more than 15 minutes in 2 hours.
1.0	Moderate Shock — not more than 15 minutes in 2 hours. Uniform — not more than 10 hours per day.
1.25	Moderate Shock — not more than 10 hours per day. Uniform — more than 10 hours per day.
1.50	Heavy Shock — not more than 15 minutes in 2 hours. Moderate Shock — more than 10 hours per day.
1.75	Heavy Shock — not more than 10 hours per day.
2.0	Heavy Shock — more than 10 hours per day.

Figure 2: Service factor for bevel gears

$$T = \frac{P_i}{\omega_i}$$

Equation 1: Torque equation

$$T_{in} = \frac{3 \times 745.7}{750 \times \frac{2\pi}{60}} = \mathbf{28.48 \text{ Nm}}$$

$$T_{out} = \frac{3 \times 745.7}{75 \times \frac{2\pi}{60}} = \mathbf{284.84 \text{ Nm}}$$

Using the equation that relates the number of teeth to the rotational speed ratio, we can first determine the overall speed reduction ratio. Subsequently, we can calculate the number of teeth required for each stage of reduction.

$$\frac{t_1}{t_2} = \frac{N_2}{N_1}$$

Equation 2: Teeth ratio

Reduction ratio:

$$\frac{\omega_{in}}{\omega_{out}} = \frac{750}{75} = 10$$

Given the required gear ratio of 10:1, we decided to use a two-stage speed reducer.

- 1st stage:

$$e_1 = \frac{10}{3} : 1 \approx 3.\overline{33}$$

- 2nd stage:

$$e_2 = 3 : 1 = 3$$

Combination: $3 \times 3.33 = 9.99 \approx 10$

Based on the catalog, we observe that bevel gears are available only in 2:1, 3:1, and 4:1 ratio. Given this constraint, we have chosen a 3:1 ratio for the second stage. To achieve the overall reduction of 10:1, we can then adapt the first stage accordingly to approximate the desired final reduction. The minimal teeth number for pinion (N_p) can be found with the following equation:

$$N_p = \frac{2k}{(1 + 2m)\sin^2(\phi)} \times (m + \sqrt{m^2 + (1 + 2m)\sin^2(\phi)})$$

Equation 3: Pinion minimal teeth

where:

- k = teeth coefficient
- ϕ = pressure angle
- m = gear ratio

$$N_p = \frac{2(1)}{(1+2(3.162))\sin^2 20} (3.162 + \sqrt{3.162^2 + (1 + 2(3.162)\sin^2 20)}) = 8.9 = 9 \text{ teeth}$$

With a minimum number of teeth for the pinion set at 9, we aim to achieve a gear ratio of approximately 3.162 to ensure our gearbox is as compact as possible. These ratios are also based on the availability of gears in the catalog. To achieve this combination of ratios, the number of teeth is assumed to be as follows:

Table 3: Number of teeth assumption

N_i	1	2	3	4
Number of teeth	15	50	15	45

After determining the gear reduction ratio, we needed to select gears that would meet the input/output requirements. Bevel gears emerged as the most suitable choice for our application, given their capability to transmit power between two intersecting axes at a right angle.

Among the different types of bevel gears, we considered both straight bevel gears and spiral bevel gears. Straight bevel gears are easier and less costly to manufacture due to their simpler teeth

design. However, they are less efficient in transmitting higher torques. On the other hand, spiral bevel gears, with their curved teeth, can handle higher torques more effectively. Their design ensures a more even distribution of loads, which enhances durability and longevity, making them ideal for applications requiring reliable performance over time [1].

To make an informed decision, we conducted a detailed analysis of both gear types and used a decision matrix that evaluated factors such as cost, torque capacity, noise level, durability, efficiency, and maintenance requirements. This comprehensive approach ensured that our final selection was based on a balanced consideration of all relevant factors, aligning with the project's goals of efficiency, reliability, and cost-effectiveness.

The decision matrix used for our analysis is as follows.

Table 4: Decision matrix analysis for spiral or straight gears

Factor	Mounting requirement	Efficiency	Reliability	Noise	Torque capacity	Price	Max ratio	Total
Spiral	4	5	5	4	3	2	1	89
Straight	4	4	4	2	5	5	5	110
Weight factor	3	4	5	2	5	2	5	

Based on this decision matrix, straight bevel gears were determined to be the superior choice for our application. Therefore, based on the same criteria we can choose to have spur gear.

Table 5: Gear specification

Gear Specifications	1	2	3	4
Material	STEEL	STEEL	STEEL	CAST IRON

Catalogue	YF15	YF50A	PA7536Y-P	PA7536Y-G
Type	Spur	Spur	Bevel	Bevel
Diametral Pitch	10	10	6	6
Face	1.25	1.25	1.08	1.08
Teeth	15	50	15	45
Ratio	3	10	3	9
Transverse Diametral Pitch	1.5	5	2.5	7.5
Bore	0.75	1	0.875	1.125
Press. Angle	20	20	20	20
Hub Dia.	1.22	3.95	2.12	3.25
Hub Proj.	0.62	0.88	1.44	1.25
Outside Diameter	1.7	5.2	2.95	7.56
Mounting Distance	5.25	5.25	5.25	3
Overall Width	1.87	2.13	2.132	2.575
Keyway Size	-	-	-	3/16*3/32
Cost (CAD)	66.29	231.3	147.8	371.4
ID #	46175	10640	12522	12520
Material Selection	Grade 2 Steel Hardened	Grade 2 Steel Hardened	Grade 2 Steel Hardened	Grade 50 Cast Iron

a) First stage of reduction: Spur gears selection

i. Speed calculation

$$e_1 = \frac{\omega_1}{\omega_2} = \frac{750}{\omega_2} = 3.\overline{33}$$

Equation 4: Speed ratio

From equation 5, we can deduce the second angular (ω_2) speed that is equal to the third(ω_3).

$$\omega_2 = 225 \text{ rpm} = 23.56 \text{ rad/s}$$

ii. Diametrical pitch calculation

For the diametrical pitch we can use the equation below.

$$P_D = \frac{N}{P}$$

Equation 5: Diametrical pitch

$P_D = \text{gear pitch diameter}$

$P = \text{Given diametral pitch (Boston Gear)}$

$N = \text{Number of teeth}$

$$P_{D1} = \frac{15}{10} = 1.5 \text{ in/d}$$

$$P_{D2} = \frac{50}{10} = 5 \text{ in/d}$$

iii. Loads and forces

To find the transmitted load tangential we need to find the pitch line velocity with the following equation:

$$V = \frac{\pi d_i \omega_i}{12}$$

$$V_{1-2} = \frac{\pi d_1 \omega_1}{12} = \frac{\pi d_2 \omega_2}{12} = \frac{\pi(1.5) \times (750)}{12} = 294.52 \text{ ft/min}$$

Equation 6: Pitch velocity line

Therefore, we can find the transmitted tangential load:

$$F_t = 33000 \times \frac{H_p}{V}$$

Equation 7: Tangential force

$$F_{t1-2} = 33000 \times \frac{3}{294.52} = 336.14 \text{ lbf}$$

The radial force can be deduced with the tangential load and angle ϕ :

$$F_r = F_t \tan(\phi)$$

Equation 8: Radial force

$$F_{r1-2} = F_{t1-2} \tan(\phi)$$

$$F_{r1-2} = 334.14 \tan(20) = 121.62 \text{ lbf}$$

b) Second stage of reduction: Straight bevel gears selection

i. Gear ratio and angular speed calculation

From equation 5 we can find the gear ratio for the second stage.

$$e_2 = \frac{\omega_3}{\omega_4} = \frac{224}{\omega_4} = 3$$

$$\omega_4 = 75 \text{ rpm} = 7.85 \text{ rad/s}$$

ii. Diametrical pitch calculation

From equation 6 we can find the diametrical pitch for the second stage:

$$P_{D3} = \frac{15}{6} = 1.5 \text{ in/d}$$

$$P_{D4} = \frac{45}{6} = 7.5 \text{ in/d}$$

iii. Loads and forces

From equation 7 the pitch line velocity is given by:

$$V_{3-4} = \frac{\pi d_3 \omega_3}{12} = \frac{\pi d_4 \omega_4}{12} = \frac{\pi(2.5) \times (225)}{12} = 147.26 \text{ ft/min}$$

The tangential load is given by:

$$F_{t3-4} = 33000 \times \frac{3}{147.26} = 672.27 \text{ lbf}$$

And for the radial force:

$$F_{r3-4} = 672.27 \tan (20) = 244.69 \text{ lbf}$$

c) Stress calculation

Analysing the strength of gears according to AGMA standards, involves determining the bending fatigue and surface fatigue strengths. To achieve this, it is necessary to identify the values of the relevant factors to calculate the ultimate and maximum stress for each fatigue mode (S_n , S_H , σ_H , and σ). The following variables have been derived from the textbook "Fundamentals of Machine Component Design."

i. Spur gears

For the spur gears the gear-tooth-bending stress can be based on the Lewis equation below:

$$\sigma = \frac{F_t P}{b J} K_v K_o K_m$$

Equation 9: Gear tooth bending stress

Given a pressure angle of 20° and **15 teeth** and **50 teeth** we obtain a value of:

J1: From the figure 15.23, we find a geometry factor of 0.25

J_2 : From the figure 15.23, we find a geometry factor of 0.45

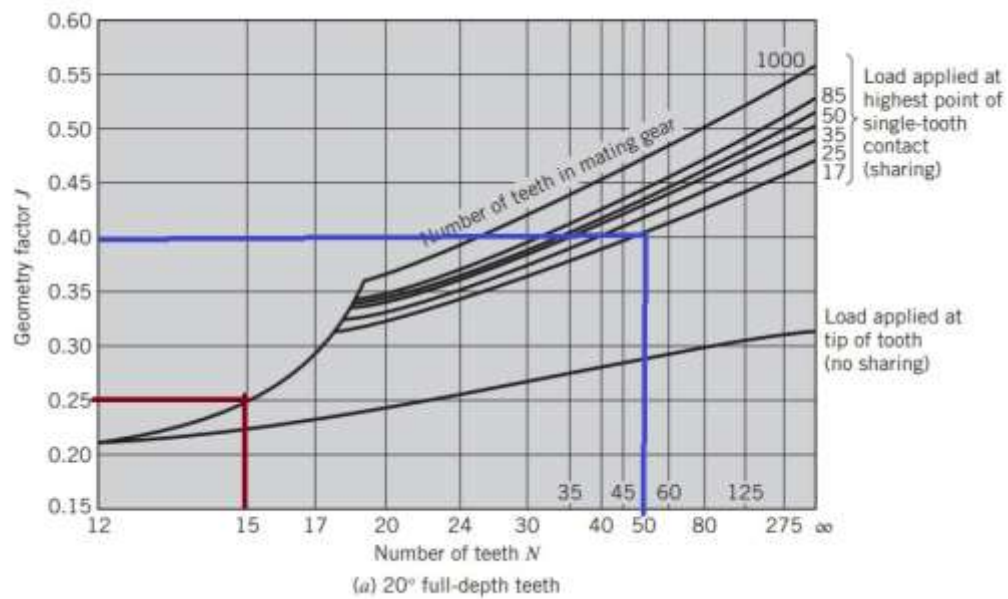


Figure 3: Geometry factor J graph for standard spur gears

For the overload correction factor K_o a moderate shock with uniform source of power is chosen for the first set of gears. We conclude that $K_o = 1.25$.

Table 15.1 Overload Correction Factor K_o

Source of Power	Driven Machinery		
	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 6: Overload Correction factor

To find the mounting factor K_m , reflecting the accuracy of mating gear alignment, we know that our face width is 1.25 therefore $K_m = 1.3$.

Table 15.2 Mounting Correction Factor K_m

Characteristics of Support	Face Width (in.)			
	0 to 2	6	9	16 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across the full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists		Over 2.2		

Table 7: Mounting correction factor

The velocity factor K_v indicates de the severity of impact as successive pairs of teeth engage. For a high precision, shaved and ground and $V=294.52 \text{ ft/min}$.

$$K_v = \frac{78 + \sqrt{V}}{78} = 1.22$$

We can find the gear bending stress with Lewis's equation:

$$\sigma_1 = \frac{(336.14)(10)}{(1.25)(0.25)} (1.22)(1.3)(1.25) = 21324.73 \text{ psi} = 21.32 \text{ ksi}$$

$$\sigma_2 = \frac{(336.14)(10)}{(1.25)(0.45)} (1.22)(1.3)(1.25) = 11847.08 \text{ psi} = 11.85 \text{ ksi}$$

Finding the geometry factor I using the following equations gives:

$$I = \frac{\sin(20) \cos(20)}{2} \cdot \frac{\frac{d_g}{d_p}}{\frac{d_g}{d_p} + 1} = \mathbf{0.124}$$

Equation 10: Geometry factor for spur gears

Following the recommended procedure from the class textbook, the elastic coefficient is evaluated using the formula provided. Given that both the pinion (p) and gear (g) are made of the same material, their Poisson's ratio and Young's modulus are identical. Therefore, the elasticity coefficient is:

Table 15.4a Values of Elastic Coefficient C_p for Spur Gears, in $\sqrt{\text{psi}}$ (Values Rounded Off)

Pinion Material ($\nu = 0.30$ in All Cases)	Gear Material			
	Steel	Cast Iron	Aluminum Bronze	Tin Bronze
Steel, $E = 30,000$ ksi	2300	2000	1950	1900
Cast iron, $E = 19,000$ ksi	2000	1800	1800	1750
Aluminum bronze, $E = 17,500$ ksi	1950	1800	1750	1700
Tin bronze, $E = 16,000$ ksi	1900	1750	1700	1650

Table 8: Values of elastic coefficient

$$C_p = 2300$$

The yields CR is calculated with:

$$CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c \sin \phi}{p_b}$$

$$CR = 0.317$$

Equation 11: Yield equation

For Bending loads: $C_L = 1$; $C_G = 1$; finally we find $C_s = 0.79$ with :

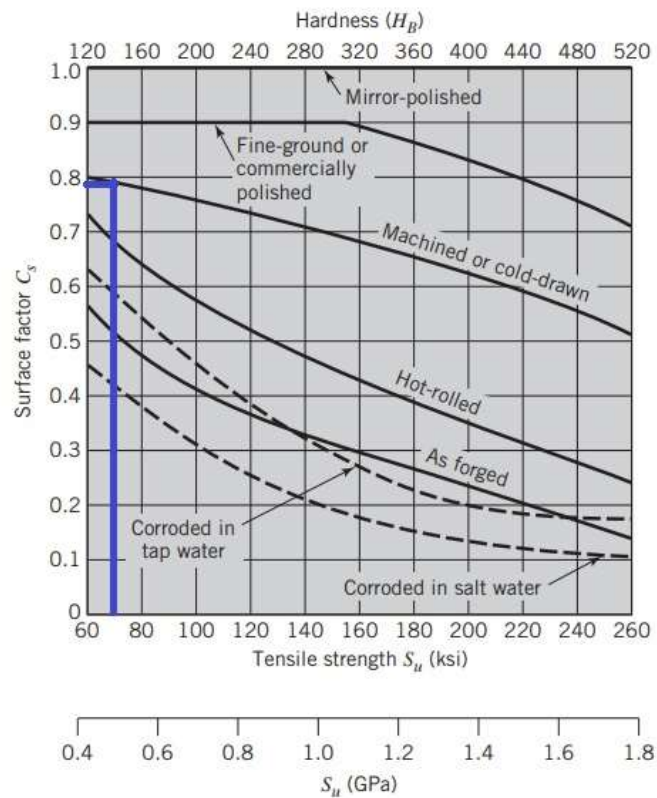


Figure 4: Surface factor

We can find the reliability correction factor $K_r=0.753$:

Reliability (%)	50	90	99	99.9	99.99	99.999
Factor k_r	1.000	0.897	0.814	0.753	0.702	0.659

Table 9: Reliability correction factor

We know that we have input gears so mean stress factor: $k_{ms}= 1.4$

Before finding the temperature factor we need to assume a working average temperature of 180°F
therefore:

$$k_t = \frac{620}{460 + T} = \frac{620}{460 + 180} = 0.96875 \text{ (for } T > 160^\circ\text{F)}$$

Equation 12: Temperature factor

CLi: Like mentioned previously since the amount of life cycle is 9.5×10^5 , we find in the figure 15.27 of the class textbook that the values of our surface fatigue life are approximately **1.15**.

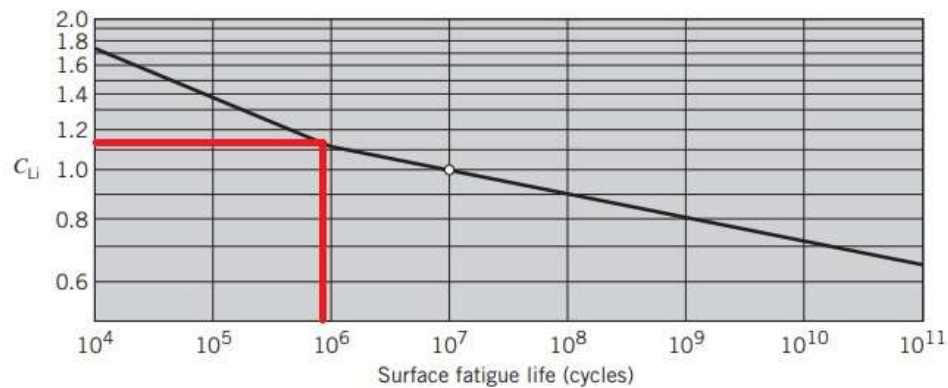


Figure 5: Values of CLi

Cr: For the reliability factor, since we want our system to have the maximum reliability, we will select a percentage of 99% reliability. This will give us a **Cr =1.00**

Reliability (%)	C_R
50	1.25
99	1.00
99.9	0.80

Table 10: Reliability factor

Sfe: From the table 15.5 of the class textbook, we find that value for surface fatigue strength (Sfe) can be found using the equation: $Sfe = 0.4 \times (Bhn) - 10kpi$. Since we already have the value for HB of the AISI 4140 steel of 302, we can go ahead and calculate the value of **Sfe = $0.4 \times (302) - 10 = 110.8 \text{ ksi}$**

Table 15.5 Surface Fatigue Strength S_{fs} , for Use with Metallic Spur Gears
(10^7 -Cycle Life, 99% Reliability, Temperature < 250°F)

Material	S_{fs} (ksi)	S_{fs} (MPa)
Steel	0.4 (Bhn)-10 ksi	2.8 (Bhn)-69 MPa
Nodular iron	0.95[0.4 (Bhn)-10 ksi]	0.95[2.8 (Bhn)-69 MPa]
Cast iron, grade 20	55	379
grade 30	70	482
grade 40	80	551

Table 11: Surface fatigue strength

S_n' : The Boston Gear catalog indicates the use of untreated 0.4 carbon normalized steel for their metal gears but does not specify the exact AISI steel grade. Based on Appendix C-a4 of the class textbook, we can infer that the 0.4 carbon steel likely falls within the 4000 series. Assuming a common industry choice for gears, it is reasonable to consider AISI 4140 as the material. Consequently, the standard fatigue strength for rotating/bending is calculated as follows:

$$S_n' = 0.5 \times S_u = 0.5 \times 148 = 74.0 \text{ ksi}$$

Designation	Factor	Value
Geometry I	I	0.124
Geometry J	J	0.25
Contact ratio	CR	0.317
Velocity	Kv	1.22
Overload	Ko	1.25
Mounting	Km	1.3
Elastic Coefficient	Cp	2300
Load	C_L	1
Gradient	C_G	1
Surface	C_S	0.79

Reliability	K_r	0.753
Temperature	K_T	0.96875
Mean stress	K_{ms}	1.4
Surface fatigue life	C_{Li}	1.15
Reliability (strength)	C_R	1
Fatigue strength for rotating/bending	S'_n	74
Surface fatigue strength	S_{fe}	110.8 ksi

Table 12: Recap of all factors for spur gears

We can now proceed to the calculation of our stress and bending fatigue including their respected endurance limit. Note that the variable b is the face width of a gear. Using the formula provided in our class material, we find that the four equations are the following:

$$S_H = S_{fe} C_{Li} C_R$$

Equation 13: Surface endurance limit

$$\sigma_H = C_p \sqrt{\frac{F_t}{bd_p I} K_v K_o K_m}$$

Equation 14: Spur gear surface fatigue stress

$$S_n = S'_n C_L C_G C_S K_r K_t K_{ms}$$

Equation 15: Bending endurance limit

After substitution of variables inside those equations, we evaluate the following equations:

Value	Operation	Result
S_H	(110800) (0.89)(1)	98.61ksi
σ_H	$2300 \sqrt{\frac{336.14}{(1.25)(1.5)(0.124)}} (1.22)(1.25)(1.3)$	123.13ksi
S_n	(74)(0.75)(0.753)(0.56875)(1.4)	33.28ksi

Table 13: Stress, endurance and surface limits

Finally, we can find the factor of security for our first set of gears:

Security factor	Result
Bending fatigue: $FOS = \frac{S_n}{\sigma}$	1.57
Surface fatigue: $FOS = \frac{S_H}{\sigma_H}$	1.25

Equation 16: Security factors for spur gears

ii. Straight bevel gears:

To determine the safety factor for straight bevel gears, we must follow the same protocol used for spur gears. This involves identifying and calculating all the types of stresses exerted on the gear teeth. These stresses include bending stress, surface fatigue stress, and other relevant factors. By thoroughly analyzing these stresses, we can ensure that the straight bevel gears are designed to withstand operational loads and achieve a satisfactory level of reliability and durability. This process helps in predicting potential gear failures and enhances the overall performance and longevity of the gear system.

Strating by finding J factor knowing that N=15 teeth we find **J=0.23**:

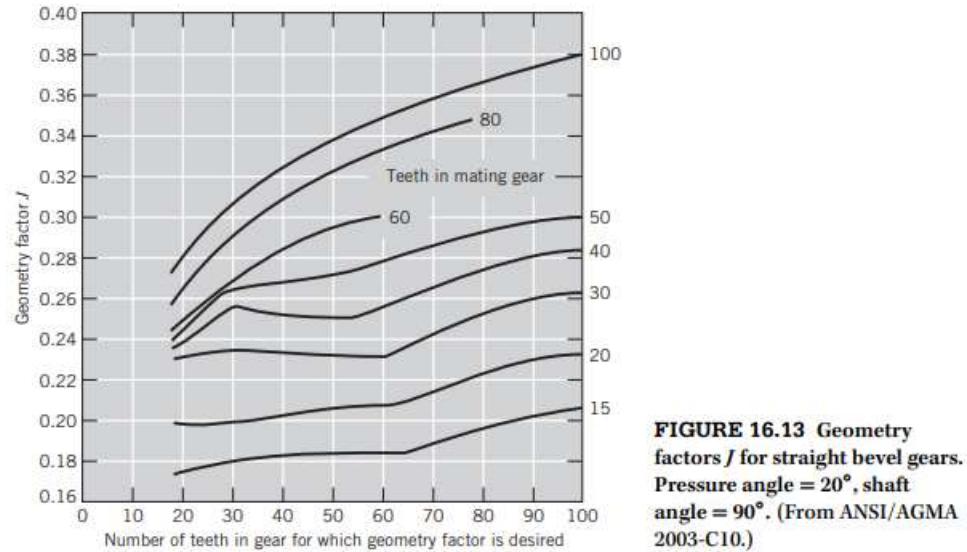


Figure 6: J factor for straight bevel gear

Finding K_v is the same process as spur gears:

$$K_v = \sqrt{\frac{78 + \sqrt{V}}{78}} = \sqrt{\frac{78 + \sqrt{142.26}}{78}} = 1.075$$

To find the mounting factor we need to follow a value for a certain mounting type for bevel gears, therefore we have a value of $K_m=1.16$.

Table 16.1 Mounting Factor K_m for Bevel Gears		
Mounting Type		Mounting Rigidity, Maximum to Questionable
Both gears straddle-mounted		1.0 to 1.25
One gear straddle-mounted; the other overhung		1.1 to 1.4
Both gears overhung		1.25 to 1.5

Figure 7: K_m factor for bevel gear

Recalling the value of $b=2.132$, we can determine that $K_o=1.25$, which is consistent with the value used for spur gears operating with a uniform power source and under moderate shock conditions. With this value established, we can proceed to calculate σ using the same equation as previously applied for the spur gears. This ensures consistency in our calculations and allows for an accurate assessment of the stresses involved.

$$\sigma = \frac{672.27(1.5)}{2.132(0.23)}(1.075)(1.25)(1.16) = 3205.5 \text{psi} = 3.2 \text{ksi}$$

We assumed that we have steel AISI 4140 gears we can find $S'_n=0.5S_u=0.5(148)=74 \text{ ksi}$.

Using the same tables and figures as before for bending loads $C_L = 1.0$; $C_G = 0.85$ for $P < 5$; $K_r = 0.814$ for 99% of reliability ; $K_t = 1.0$; for $K_{ms} = 1.4$ for input and output; $C_S = 0.7$

$$S_H = S_{fe} C_{Li} C_R$$

$$S_{fe} = 0.4 (bhn) - 10 \text{ksi} = 110.8 \text{ ksi}$$

$$C_{Li} = 1.0 ; C_R = 1.0$$

$$C_p=2300; I=0.12358; K_o=1.25; K_v=1.13328; d_p=6 ; F_t=672.27 ; K_m=1.16$$

Designation	Factor	Value
Geometry I	I	0.12358
Geometry J	J	0.23
Velocity	K_v	1.13328
Overload	K_o	1.25
Mounting	K_m	1.16
Elastic Coefficient	C_p	2300
Load	C_L	1
Gradient	C_G	0.85
Surface	C_S	0.79

Reliability	K_r	0.814
Temperature	K_T	1
Mean stress	K_{ms}	1.4
Surface fatigue life	C_{Li}	1
Reliability (strength)	C_R	1
Fatigue strength for rotating/bending	S'_n	74

Table 14: Recap of factors

$$S_n = 74 \times 1.0 \times 0.85 \times 0.7 \times 0.814 \times 1.0 \times 1.4 = 50.176$$

$$S_H = 110.8$$

$$\sigma_H = 60799.9 \text{ psi} = 60.8 \text{ ksi}$$

Bending fatigue: $FOS = \frac{S_n}{\sigma}$	15.68
Surface fatigue: $FOS = \frac{S_H}{\sigma_H}$	1.82

Table 15: Bevel gears security factor

4. Shaft Analysis

The shaft analysis consists of calculating the stresses that are applied to the shafts. Both bending loads and torsional loads are considered. When the shafts are transmitting power, they are subject to torque which is accompanied by alternating bending stresses due to the gears. Stress concentration factors need to be found for each change in the diameter of the shaft. A fatigue strength calculation will then help in finding S_n .

For the shafts, the material chosen was AISI 4140 which possesses good characteristics with a tensile strength $S_u = 148 \text{ ksi}$ and $S_y = 95 \text{ ksi}$.

The location at which bending moments occur is important for drawing FBDs, shear and moment diagrams for both vertical and horizontal planes. It is assumed that the acceleration is zero and that transversal shear stresses are negligible compared to bending and torsional stresses. The Goodman criterion is used to find the safety factor and to ensure that it is respected, which will ensure that the design is robust and safe throughout the life cycle of the gear reducer.

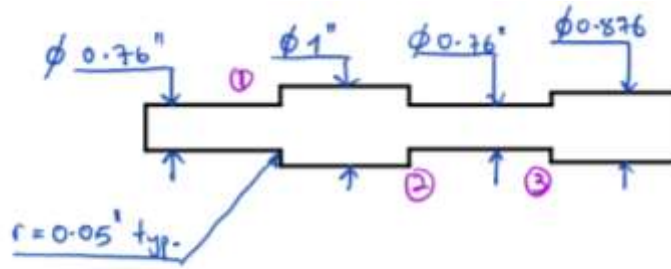
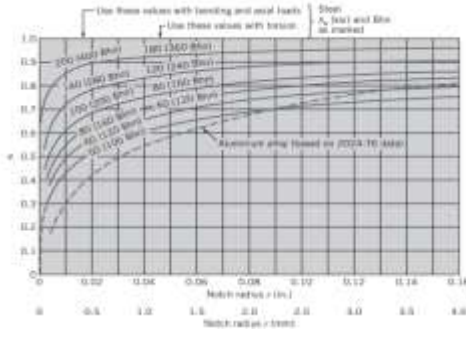


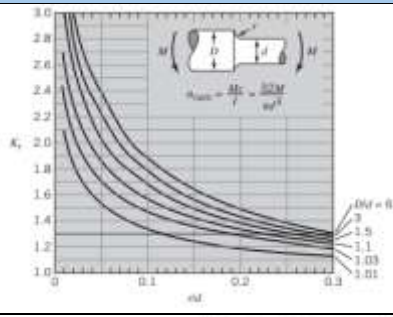
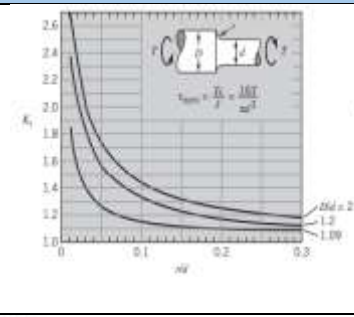
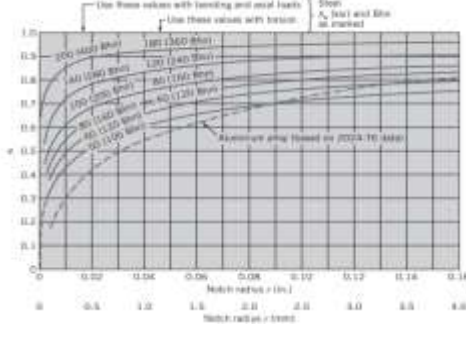
Figure 8: Intermediate shaft dimensions

For section 1 and 2: $\frac{D}{d} = 1.316$; $\frac{r}{d} = 0.066$

Factor	Type of load		Figure and/or formula	
	Bending	Torsion	Bending	Torsion
Kt	1.9	1.5		

q	0.87	0.9	
K_f	1.78	1.45	$1+(K_t-1)q$

For section 3: $\frac{D}{d} = 1.15$; $\frac{r}{d} = 0.057$

Factor	Type of load		Figure and/or formula	
	Bending	Torsion	Bending	Torsion
Kt	1.8	1.55		
q	0.87	0.9		
K_f	1.7	1.5	$1+(K_t-1).q$	

Stress analysis		
$S_n = S_n' C_L C_G C_S K_r K_t K_{ms}$		
Bending stress		Torsional stress
S_n'	0.5 Su=74 ksi	0.5 Su= 74 ksi
C_L	1.0	0.58
C_G	0.9	0.9
C_S	0.7 (cold-drawn)	0.7 (cold-drawn)
C_T	1.0	1.0
C_r	1.0 (50% reliability)	1.0 (50% reliability)
S_n	46.62	27.04

Table 16: Stress analysis for intermediate shaft

Critical Torsional Analysis			
$\tau_{nom} = \frac{16T}{\pi d^3} K_f ; \quad \text{with } T = \text{input torque} = 252.07 \text{ lbin}$			
Section:	1	2	3
d	0.76"	1"	0.876"
K_f	1.45	1.45	1.5
τ_{nom}	4.24 ksi	1.86 ksi	2.86 ksi

Table 17: Critical torsional analysis for intermediate shaft

Critical Bending Analysis:

$$s_{nom} = \frac{32 M}{\pi d^3} K_f$$

$$\begin{cases} F_{r1-2} = 121.62 \text{ lbf} \\ F_{r3-4} = 244.69 \text{ lbf} \end{cases}$$

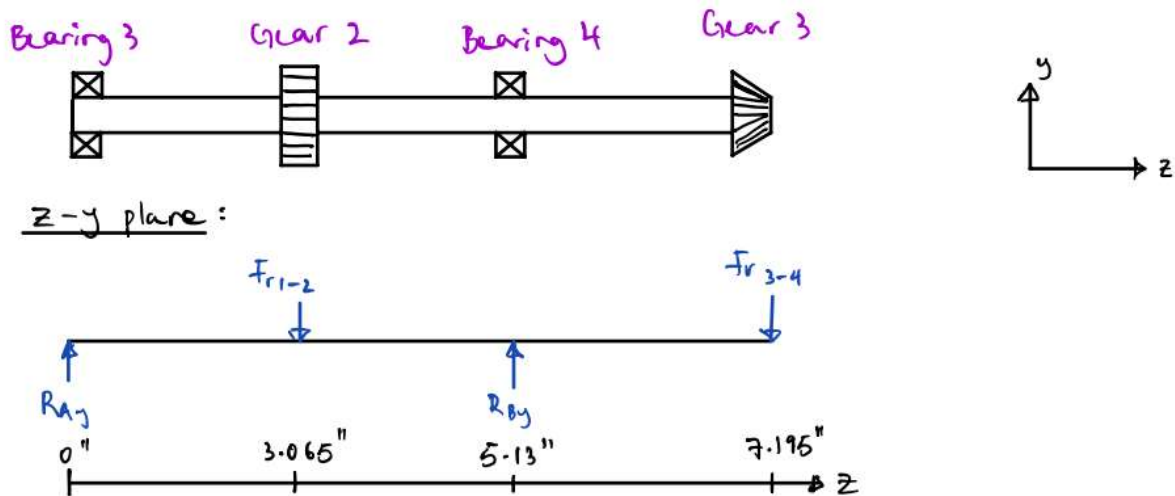


Figure 9: Intermediate shaft critical bending analysis

$$+\uparrow \Sigma F_y : R_{Ay} + R_{By} - F_{r1-2} - F_{r3-4} = 0$$

$$R_{Ay} + R_{By} - 121.62 - 244.69 = 0$$

$$R_{Ay} + R_{By} = 366.31 \quad \text{--- (1)}$$

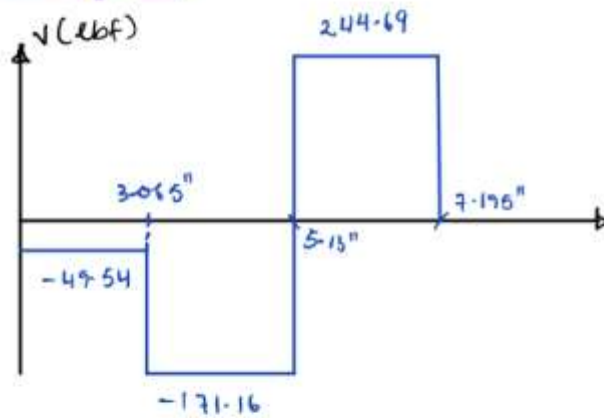
$$(\uparrow \Sigma M_A = 0 : R_{By}(5.13) - F_{r1-2}(3.065) - F_{r3-4}(7.195) = 0$$

$$R_{By}(5.13) = 2133.31$$

$$R_{By} = 415.85 \text{ lbf}$$

$$\therefore R_{Ay} = -49.54 \text{ lbf } (\downarrow)$$

V-Diagram :



M-Diagram

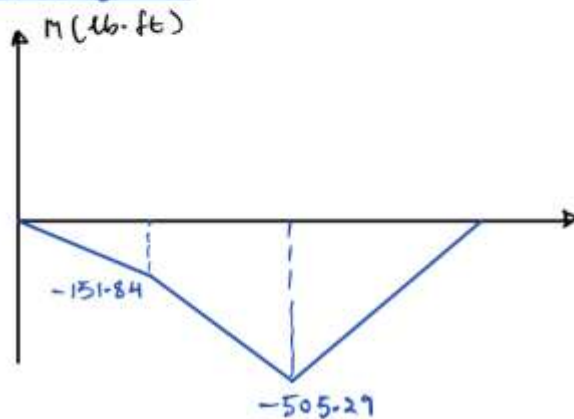
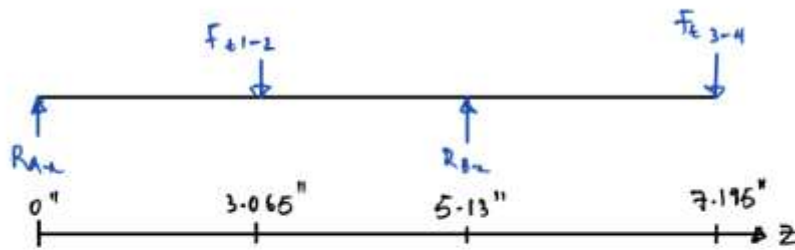


Figure 10: Intermediate shaft V & M diagrams

z-x plane:



$$\begin{cases} F_{t1-2} = 336.14 \text{ lbf} \\ F_{t3-4} = 672.27 \text{ lbf} \end{cases}$$

$$+\uparrow \Sigma F_y: R_{Ax} + R_{Bz} - F_{t1-2} - F_{t3-4} = 0$$

$$R_{Ax} + R_{Bz} - 336.14 - 672.27 = 0$$

$$R_{Ax} + R_{Bz} = 1008.41 \quad \text{--- ①}$$

$$\downarrow \Sigma M_A: R_{Bz}(5.13) - F_{t1-2}(3.065) - F_{t3-4}(7.175) = 0$$

$$R_{Bz}(5.13) = 5867.25$$

$$R_{Bz} = 1143.71 \text{ lbf}$$

$$\rightarrow \text{①: } R_{Ax} = -135.3 \text{ lbf} \quad (\downarrow)$$

V-Diagram:

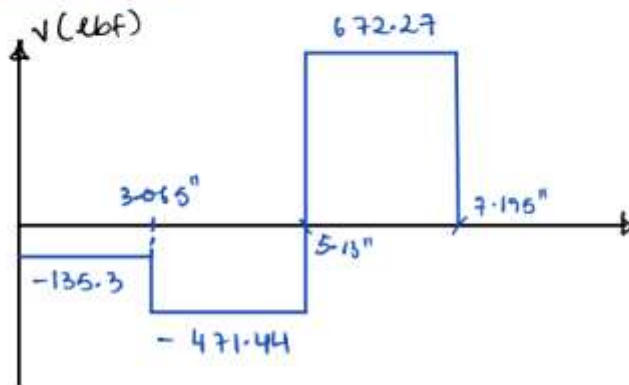
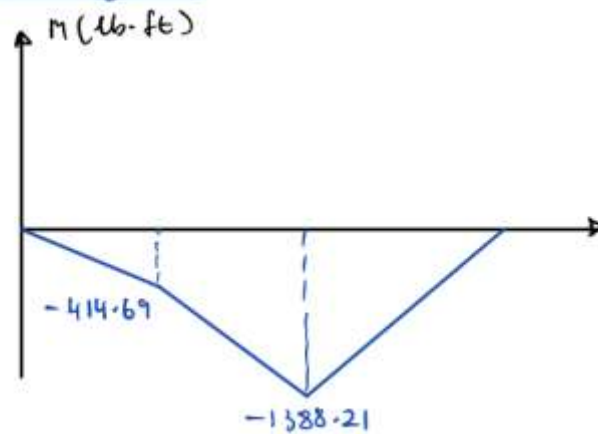


Figure 11: Intermediate shaft forces calculation

M-Diagram



$$M = \sqrt{M_{z-x}^2 + M_{z-y}^2} = \sqrt{(-1388.21)^2 + (-505.29)^2} = 1477.31 \text{ lbin}$$

①: $d = 0.76''$, $K_f = 1.45$

$$S_{nom_1} = \frac{(32)(1477.31)(1.45)}{\pi(0.76)^3} = 49704.87 \text{ psi} = 49.7 \text{ ksi}$$

②: $d = 1''$, $K_f = 1.45$

$$S_{nom_2} = \frac{(32)(1477.31)(1.45)}{\pi(1)^3} = 21819.2 \text{ psi} = 21.8 \text{ ksi}$$

③: $d = 0.876''$, $K_f = 1.50$

$$S_{nom_3} = \frac{(32)(1477.31)(1.50)}{\pi(0.876)^3} = 33577.7 \text{ psi} = 33.6 \text{ ksi}$$

Figure 12: Intermediate shaft nominal stresses

Equivalent bending stress:

$$\sigma_{ea} = \sqrt{\sigma_a^2 + 3\tau_a^2}$$

$$\tau_a = 0, \quad \sigma_a = \sigma_{nom} = 49.7 \text{ ksi}$$

$$\therefore \sigma_{ea} = 49.7 \text{ ksi}$$

Equivalent mean bending stress:

$$\sigma_{em} = \frac{\sigma_m}{2} + \sqrt{\tau_m^2 + \left(\frac{\sigma_m}{2}\right)^2}$$

$$\sigma_m = 0, \quad \tau_m = \tau_{nom} = 4.24 \text{ ksi}$$

$$\therefore \sigma_{em} = 4.24 \text{ ksi}$$

Modified Goodman Criterion with safety factor n :

$$\frac{\sigma_{ea}}{S_n} + \frac{\sigma_{em}}{S_u} = \frac{1}{n}$$

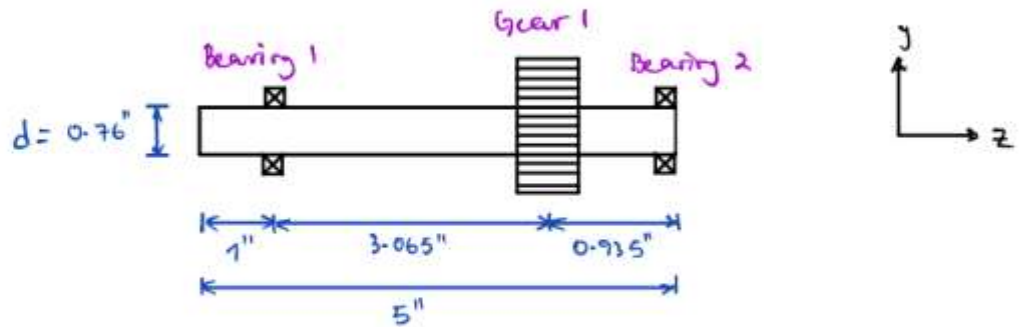
$$\frac{49.7}{46.62} + \frac{4.24}{148} = \frac{1}{n}$$

$$n = 1.1 \checkmark \text{ ok}$$

Figure 13: Intermediate shaft security factor calculation

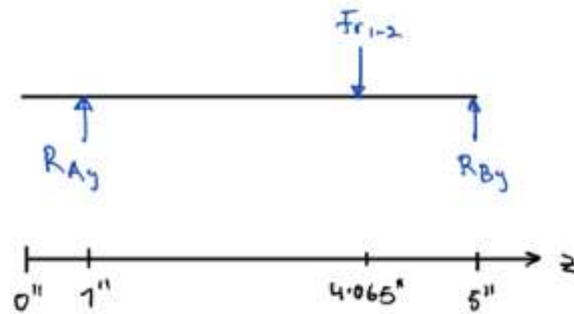
Input Shaft

Critical Bending Analysis:



z - y plane

$$F_{r1-2} = 121.62 \text{ lbf}$$



$$+\uparrow \Sigma F_y: R_{Ay} + R_{By} - F_{r1-2} = 0$$

$$R_{Ay} + R_{By} = 121.62 \text{ lbf}$$

$$(\downarrow \Sigma M_B: -R_{Ay}(4) + F_{r1-2}(1.065) = 0$$

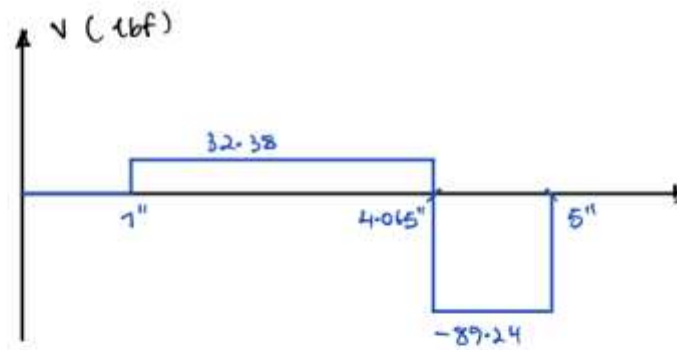
$$-R_{Ay}(4) + 121.62(1.065) = 0$$

$$\rightarrow R_{Ay} = 32.38 \text{ lbf}$$

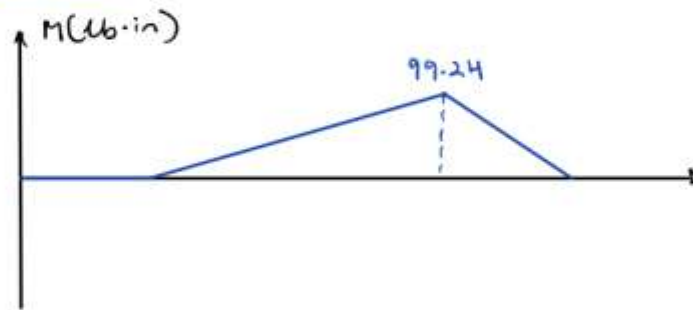
$$\rightarrow R_{By} = 89.239 \text{ lbf}$$

Figure 14: Input shaft critical bending analysis

V-Diagram:

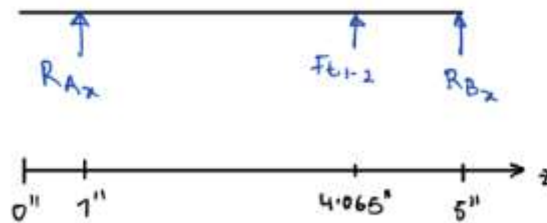


M-Diagram:



z-x plane

$$F_{t1-2} = 336.14 \text{ lbf}$$



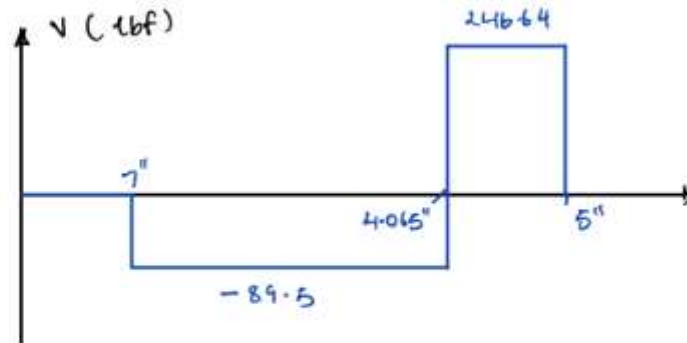
$$\uparrow \sum F_z = 0 : R_{Ax} + R_{Bx} = -336.14 \text{ lbf}$$

$$\begin{aligned} \downarrow \sum M_B : -R_{Ax}(4) - F_{t1-2}(1.065) &= 0 \\ -R_{Ax}(4) - 336.14(1.065) &= 0 \end{aligned}$$

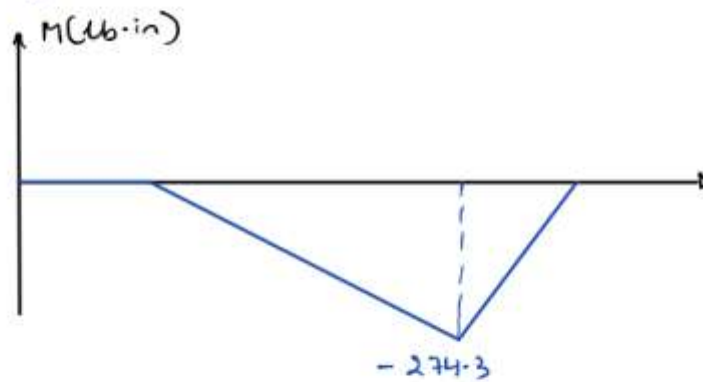
Figure 15: Input shaft V & M diagrams

$$\rightarrow R_{A_z} = -89.5 \text{ lbf } (\downarrow)$$

$$\rightarrow R_{B_z} = -246.64 (\downarrow)$$



M - Diagram:



$$M = \sqrt{M_{z-x}^2 + M_{z-y}^2} = 291.7 \text{ lb-in}$$

$$\begin{aligned} \sigma_{nom} &= \frac{32M}{\pi d^3} = \frac{(32)(291.7)}{\pi (0.76)^3} = 6768.55 \text{ psi} \\ &= 6.77 \text{ ksi} \end{aligned}$$

Critical torsional analysis

$$\begin{aligned} \tau_{nom} &= \frac{16T}{\pi d^3} = \frac{16(252.07)}{\pi (0.76)^3} = 2924.49 \text{ psi} \\ &= 2.9 \text{ ksi} \end{aligned}$$

Figure 16: Input shaft critical torsional analysis

Equivalent bending stress:

$$\sigma_{ea} = \sqrt{\sigma_a^2 + 3\tau_a^2}$$

$$\tau_a = 0, \quad \sigma_a = \sigma_{nom} = 6.77 \text{ ksi}$$

$$\therefore \sigma_{ea} = 6.77 \text{ ksi}$$

Equivalent mean bending stress:

$$\sigma_{em} = \frac{\sigma_m}{2} + \sqrt{\tau_m^2 + \left(\frac{\sigma_m}{2}\right)^2}$$

$$\sigma_m = 0, \quad \tau_m = \tau_{nom} = 2.9 \text{ ksi}$$

$$\therefore \sigma_{em} = 2.9 \text{ ksi}$$

Modified Goodman Criterion with safety factor n :

$$\frac{\sigma_{ea}}{S_n} + \frac{\sigma_{em}}{S_u} = \frac{1}{n}$$

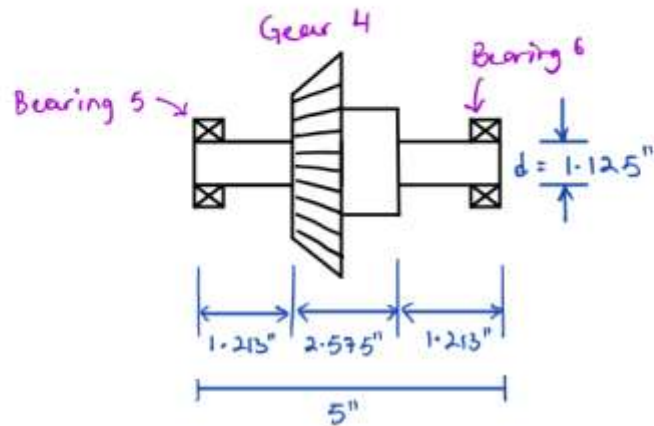
$$\frac{6.77}{46.62} + \frac{2.9}{148} = \frac{1}{n}$$

$$n = 6.1 \quad \checkmark \quad \text{OK}$$

Figure 17: Input shaft security factor

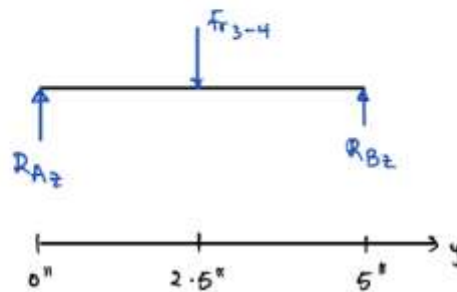
Output shaft

Critical bending Analysis



y-z plane

$$F_{r3-4} = 244.69 \text{ lbf}$$

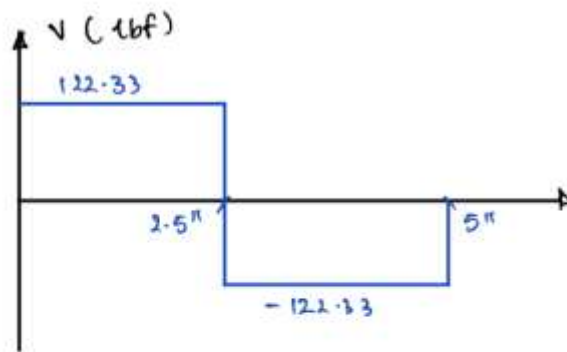


$$\begin{aligned} \uparrow \sum F_z = 0: \quad R_{Az} - F_{r3-4} + R_{Bz} &= 0 \\ R_{Az} + R_{Bz} &= 244.69 \text{ lbf} \end{aligned}$$

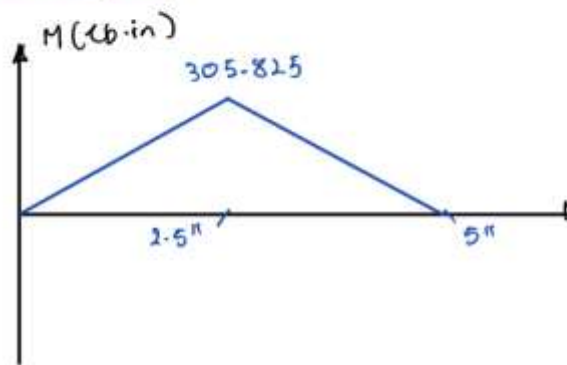
$$\begin{aligned} \downarrow \sum M_A = 0: \quad -F_{r3-4}(2.5) + R_{Bz}(5) &= 0 \\ R_{Bz}(5) &= 244.69(2.5) \\ \rightarrow R_{Bz} &= 122.33 \text{ lbf} \\ \rightarrow R_{Az} &= 122.33 \text{ lbf} \end{aligned}$$

Figure 18: Output shaft critical bending analysis

V - Diagram :

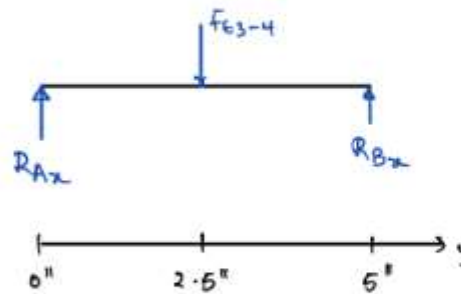


M - Diagram :



x-y plane

$$F_{t3-4} = 672.27 \text{ lbf}$$



$$\begin{aligned} \uparrow \sum F_y = 0 : \quad R_{Ax} - F_{t3-4} + R_{Bx} &= 0 \\ R_{Ax} + R_{Bx} &= 672.27 \text{ lbf} \end{aligned}$$

Figure 19: Output shaft V & M diagrams

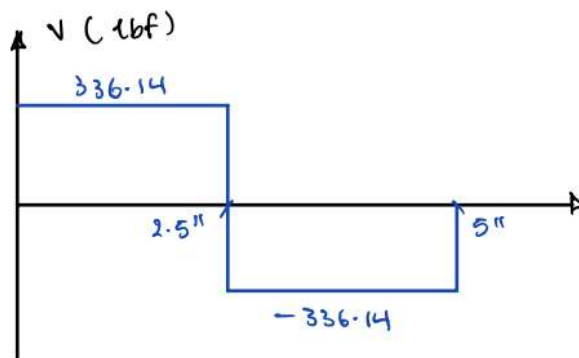
$$(\uparrow \Sigma M_A = 0 : -F_{b3-4}(2.5) + R_{Bx}(5) = 0$$

$$R_{Bx}(5) = 672.27(2.5)$$

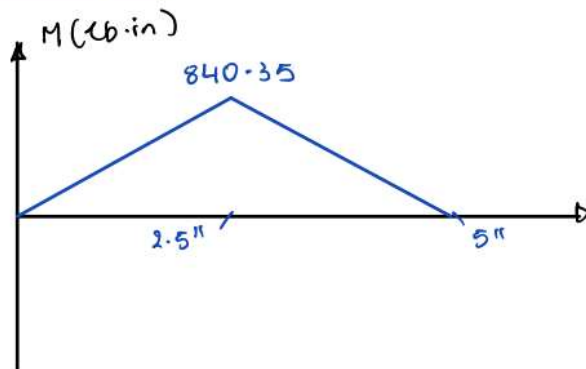
$$\rightarrow R_{Bx} = 336.14 \text{ lbf}$$

$$\rightarrow R_{Ax} = 336.14 \text{ lbf}$$

V - Diagram :



M - Diagram :



$$M = \sqrt{M_{x-y}^2 + M_{y-z}^2} = 894.27 \text{ lb-in}$$

$$s_{nom} = \frac{32M}{\pi d^3} = \frac{(32)(894.27)}{\pi(1.125)^3} = 6397.51 \text{ psi}$$

$$= 6.40 \text{ ksi}$$

Figure 20: Output shaft nominal stress calculation

Critical torsional analysis

$$T_{out} = 284.84 \text{ Nm} \\ = 2521.05$$

$$\tau_{nom} = \frac{16 T_{out}}{\pi d^3} = \frac{16 (2521.05)}{\pi (1.125)^3} = 9017.66 \text{ psi} \\ = 9.02 \text{ ksi}$$

Equivalent bending stress:

$$\sigma_{ea} = \sqrt{\sigma_a^2 + 3\tau_a^2}$$

$$\tau_a = 0, \quad \sigma_a = \sigma_{nom} = 6.40 \text{ ksi}$$

$$\therefore \sigma_{ea} = 6.4 \text{ ksi}$$

Equivalent mean bending stress:

$$\sigma_{em} = \frac{\sigma_m}{2} + \sqrt{\tau_m^2 + \left(\frac{\sigma_m}{2}\right)^2}$$

$$\sigma_m = 0, \quad \tau_m = \tau_{nom} = 9.02 \text{ ksi}$$

$$\therefore \sigma_{em} = 9.02 \text{ ksi}$$

Modified Goodman Criterion with safety factor n :

$$\frac{\sigma_{ea}}{S_n} + \frac{\sigma_{em}}{S_u} = \frac{1}{n}$$

$$\frac{6.40}{46.62} + \frac{9.02}{148} = \frac{1}{n}$$

$$n = 5.04 \checkmark \text{ OK}$$

Figure 21: Output shaft critical torsional analysis

5. Bearing Design

Bearings are essential for the good functioning of a gearbox. A bearing is a mechanical component that constrains relative motion and reduces friction between moving parts in machinery. Bearings are generally used for the reduction of friction, to support loads, for energy efficiency, for heat reduction, vibration and noise reduction, and have many other uses.

For a good gearbox, it's important to choose the right bearings. To do this, bearings analysis is necessary. With the bearing analysis we will be able to find the different parameters (radial load F_r , thrust load F_t , equivalent load F_e , life adjustment reliability factors K_r , application factor K_a , life corresponding to rated capacity L_R , life corresponding to radial load L) that will help us to make the best choice possible.

For this gearbox we will have 8 bearings, two for each gear. For each gear, one bearing will be fixed and the other one will be floating. Bearings 1 and 2 will be for gear 1, bearings 3 and 4 will be for gear 2, bearings 5 and 6 will be for gear 4.

For this project we will use ball bearings; they will be selected according to their rated capacity. For that we will need these following equations:

$$F_e = F_r \left[1 + 1.115 \left(\frac{F_t}{F_r} - 0.35 \right) \right]$$

For $0.35 < \frac{F_t}{F_r} < 10$:

$$C_{req} = F_e K_a \left(\frac{L}{K_r L_R} \right)^{0.3}$$

$$L = 750 \text{ (rpm)} * 8000 \text{ (hr)} * 60 \left(\frac{\text{min}}{\text{hr}} \right) = 3.6 * 10^8 \text{ rev}$$

Table 18: Parameters

Bearings	$\frac{F_t}{F_r}$	F_e (lbf)	K_a (form table 14.3)	K_r (form fig. 14.13)	L_R	L (revolution)
1	2.764	448.95	1.15	1	$90 * 10^6$	$3.6 * 10^8$
2	N/A (floating)	-	1.15	1	$90 * 10^6$	$3.6 * 10^8$
3	2.764	448.95	1.15	1	$90 * 10^6$	$3.6 * 10^8$
4	N/A (floating)	-	1.15	1	$90 * 10^6$	$3.6 * 10^8$
5	2.747	898.78	1.15	1	$90 * 10^6$	$3.6 * 10^8$
6	N/A (floating)	-	1.15	1	$90 * 10^6$	$3.6 * 10^8$

Table 19: Bearings selection

Bearings	C_{req} (lbf)	Bearing type	Bore (mm)	OD (mm)	W (mm)
1	782.55	200 IT	20	47	14
2	782.55	200 IT	20	47	14
3	782.55	200 IT	20	47	14
4	782.55	200 IT	20	47	14
5	1566.64	L06 XLT	30	55	13
6	1566.64	L06 XLT	30	55	13

6. Deflection and slopes (Ihab)

1) Input shaft:

The deflection of a shaft subjected to various loading conditions is a critical aspect of structural and mechanical engineering. Understanding the behavior of the shaft under load ensures that the design can withstand operational stresses without excessive bending or failure.

Theory of Beam Deflection

For a simply supported shaft with loads applied at specific points, the deflection (δ) at any point along the shaft is influenced by:

- **Load (F_{gear}):** The magnitude of the force applied by the gear.
- **Length (L):** The span of the shaft between the supports.
- **Material Properties:** Specifically, the Young's Modulus (E), which measures the stiffness of the material.
- **Moment of Inertia (I):** A geometrical property of the shaft's cross-section that affects its resistance to bending. For a circular cross-section, it is calculated as $I = (\pi \times d^4) / 64$, where d is the diameter of the shaft.

Deflection Calculation

The deflection of a simply supported shaft with specific point loads can be determined using the principle of superposition, where the deflection at any point is the sum of the deflections due to individual loads. The deflection is calculated using the following piecewise function:

1. For $x \leq a$:

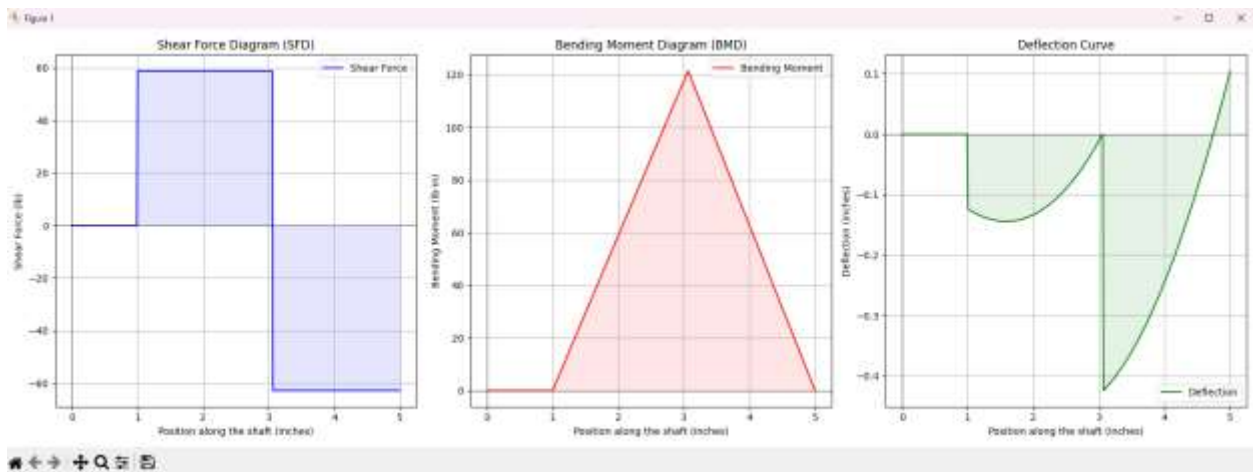
$$\delta(x) = 0$$

2. For $a \leq x \leq b$:

$$\delta(x) = \frac{R1 \cdot x^3}{6} - \frac{R1 \cdot a \cdot x^2}{2} + \frac{F_{gear} \cdot b \cdot x^2}{2} - \frac{F_{gear} \cdot b^2 \cdot x}{2} / (E \cdot I)$$

3. For $b \leq x \leq c$:

$$\delta(x) = \frac{R1 \cdot x^3}{6} - \frac{R1 \cdot a \cdot x^2}{2} - \frac{F_{gear} \cdot (x - b)^3}{6} + \frac{F_{gear} \cdot (c - b) \cdot (x - c) \cdot x}{(E \cdot I)}$$



2) Intermediate shaft

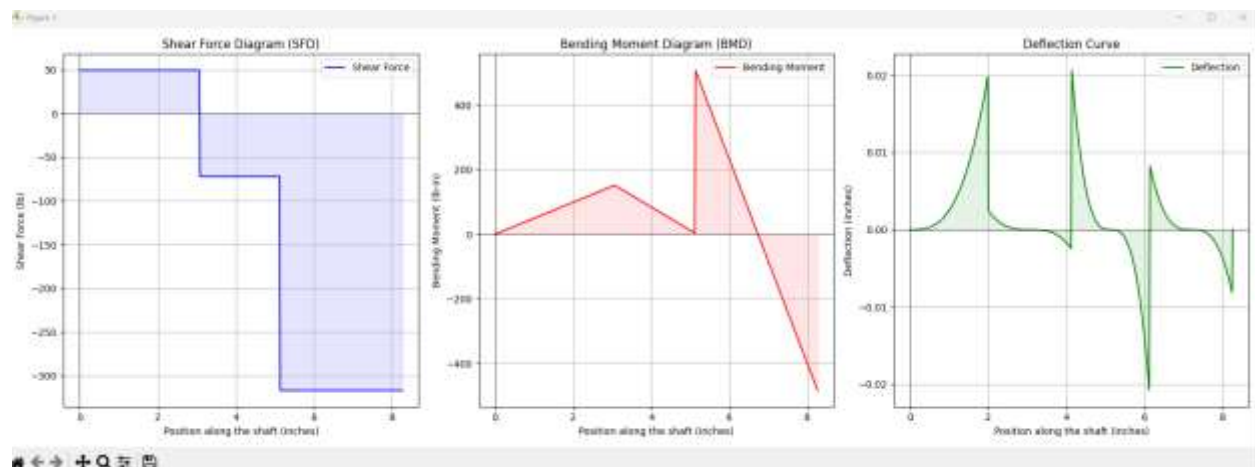
For a simply supported shaft with loads applied at specific points, the deflection (δ) at any point along the shaft is influenced by:

- **Load (F_{gear1} , F_{gear2}):** The magnitude of the forces applied by the gears.
- **Length (L):** The span of the shaft between the supports.
- **Material Properties:** Specifically, the Young's Modulus (E), which measures the stiffness of the material.

- **Moment of Inertia (I):** A geometrical property of the shaft's cross-section that affects its resistance to bending. For a shaft with varying diameters, the moment of inertia changes along its length.

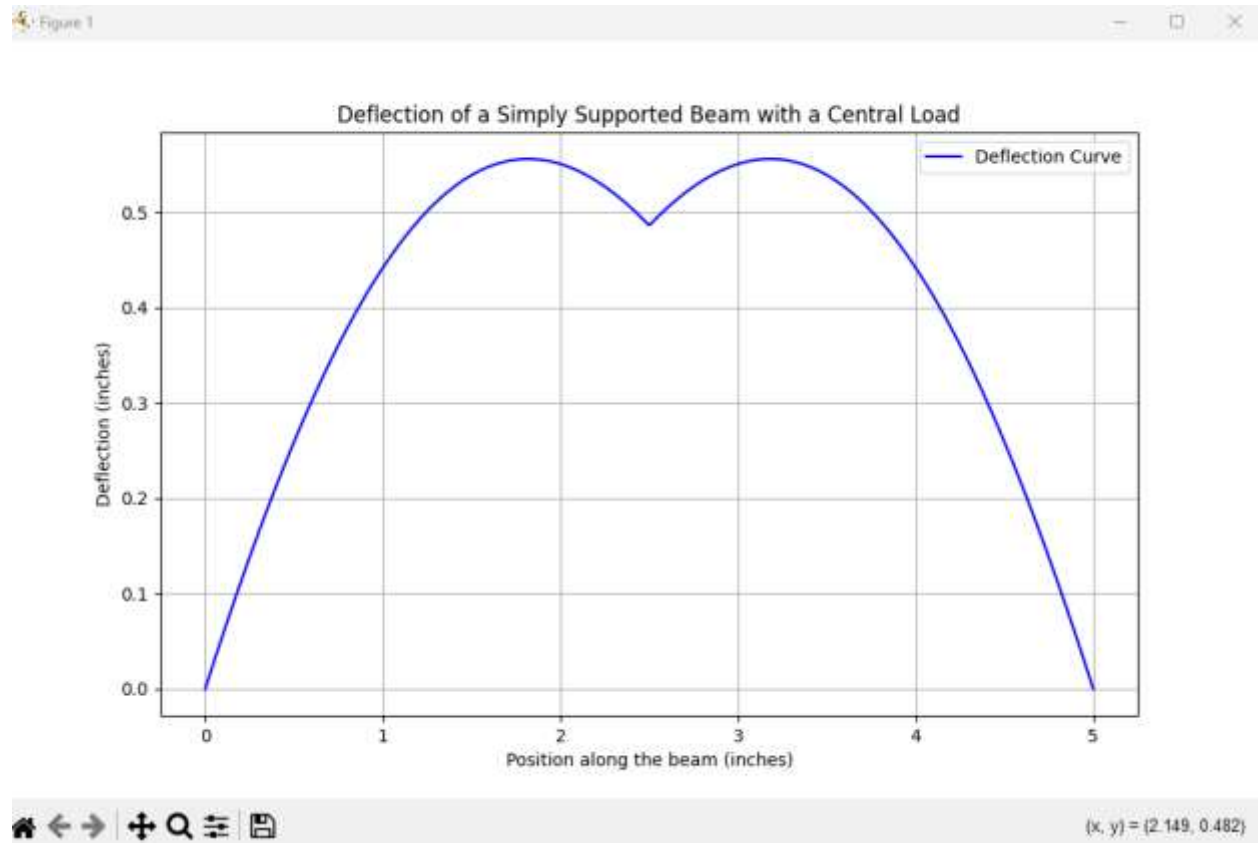
Deflection Calculation

The deflection of a simply supported shaft with specific point loads can be determined using the principle of superposition, where the deflection at any point is the sum of the deflections due to individual loads. The deflection is calculated using piecewise functions depending on the shaft's geometry and loading conditions.



3) Output shaft

Lastly following the same protocol as before we obtain a deflection diagram for the output shaft:



7. Key Design

To secure the gears on the shaft, keys and spacers are employed. In this part, we will explore the analysis of the key and the process for selecting its material. Below is a sample calculation for the key used with gear 1 mounted on the input shaft. The primary equation for this analysis is:

$$s_y = \frac{T \times 8}{0.58 \times L \times d^2}$$

Ideally, the key needs to be mounted all the way through the gear, giving $L = 1.8 \times d$. The input shaft has a diameter of 22.25mm therefore, the expected strength of the key is calculated as shown below:

$$s_y = \frac{28.48 \times 8}{0.58 \times \frac{19.3}{1000} \times \frac{40^2}{1000}} = 12.7 \text{ MPa}$$

A key made up of a material of yield strength of 12.7 MPa is needed to hold the gear together.

Since the yield strength is quite small, AISI 1015 has been chosen with a S_y of 324.1 MPa. The chosen material will be able to withstand a maximum torque of 7.74 kNm which is 270 times more than the input torque.

	Key 1 for gear 1	Key 2 for gear 2	Key 3 for gear 3	Key 4 for gear 4
L in mm	19.3	25.4	22.25	22.25
Material	AISI 1015	AISI 1015	AISI 1015	AISI 1015

8. Fits and tolerances

Gears

In general, all types of components require fits and tolerance, except for certain components. In our gearbox, we won't need to use fits and tolerances because we're using a key. The key will ensure that the gears don't slip, be fixed and therefore avoid axial forces on the gears.

Bearings

Good fits and tolerances enable bearings to optimize their performance, reduce the risk of failure and extend the life of bearings and our gearbox. For our gearbox, we're going to use a rather tight fit to avoid creep. We'll use a small tolerance of $T = \pm 50 \mu m$.

9. Critical Speed

Each shaft has a specific critical speed, which is the speed at which it becomes unstable due to unlimited increasing deflections. Identifying this critical speed is crucial for preventing shaft failure, as it serves as a benchmark. By ensuring the angular speed of each shaft remains below its critical speed, failures can be avoided. To calculate these critical speeds, we used Equation 17.1 from RC Juvinall and KM Marshek Fundamental of machine component design textbook [2]. The shafts were divided into 0.25-inch segments to simplify the calculation of weight and slope. These segment values were summed and applied in the equation, which includes variables such as g (acceleration due to gravity), w (gravitational force), δ_{st} (shaft deflection), n_c (critical speed), k (shaft spring rate) and ω_n (natural frequency).

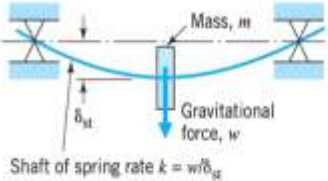
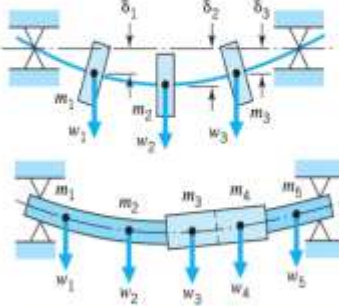
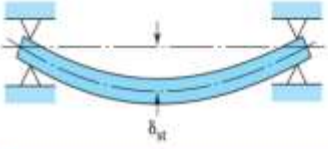
Configuration	Critical Speed Equation
<p>(a) Single mass</p>  <p>Mass, m Gravitational force, w Shaft of spring rate $k = w/\delta_{st}$</p>	$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{w}} = \sqrt{\frac{g}{\delta_{st}}} \quad (17.1)$ $n_c = \frac{30}{\pi} \sqrt{\frac{k}{m}} = \frac{30}{\pi} \sqrt{\frac{kg}{w}} = \frac{30}{\pi} \sqrt{\frac{g}{\delta_{st}}} \quad (17.1a)$
<p>(b) Multiple masses</p> 	$\left. \begin{aligned} n_c &\approx \frac{30}{\pi} \sqrt{\frac{g(w_1\delta_1 + w_2\delta_2 + \dots)}{w_1\delta_1^2 + w_2\delta_2^2 + \dots}} \\ n_c &\approx \frac{30}{\pi} \sqrt{\frac{g \sum w\delta}{\sum w\delta^2}} \end{aligned} \right\} \quad (17.2)$
<p>(c) Shaft mass only</p> 	$\omega_n \approx \sqrt{\frac{5g}{4\delta_{st}}} \quad (17.3)$ $\delta_{st} = \frac{5wL^4}{384EI}$

Figure 22: Critical speed

The actual speeds of the shaft do not even approach the critical speeds. This means the system is stable.

10. Other design concepts

Casing Design

The casing design for this gearbox, with its vertical input and horizontal output, must ensure effective shaft support, lubrication, and pressure regulation, while being cost-effective and durable.

The casing's primary function is to support the three shafts without restricting their movement. Given the need for a two-stage reduction to achieve a 10:1 ratio, the casing must ensure precise alignment and robust support for both stages. Proper alignment of the vertical input and horizontal output shafts is crucial, necessitating support at both the outer and inner ends, where the bearings will be positioned.

Another important role of the casing is to contain the oil necessary for lubrication and heat dissipation, as the design includes a splash lubrication system. The casing must ensure that oil reaches all gears and bearings without leaking. Additionally, the rise in temperature inside the housing can cause pressure buildup; hence, incorporating breather vents with filters at the top of the casing will help regulate excess pressure and prevent oil from escaping.

The material selection for the casing is essential, impacting the gearbox's strength, durability, and cost. The loads and stresses are expected to be low, and the environmental conditions are not extreme. This allows for a relatively thin wall thickness and the use of a low-grade material. Therefore, plain carbon steel has been chosen for its adequacy in meeting these demands.

Lubrication

Lubrication is a technique that uses a substance to reduce, generally frictions but also heat and wear, between two surfaces in mutual contact. Lubricants are usually liquid but can be solid, such as graphite, or gas, such as pressurized air. For our gearbox we will use liquid because it will help us to maintain the efficiency, reliability and longevity of our gear box.

For our gearbox we will use synthetic oil. It can withstand higher temperatures without breaking down, ensuring consistent performance even under extreme conditions, they can also

maintain their viscosity and flow characteristics even at low temperatures. Synthetic oil can resist oxidation and corrosion and has a very good load-carrying capacity good for our gearbox. As a synthetic oil we will use Mobil SHC oil [3].

Seals

A seal is an important component in gearboxes. Seals are used for retaining lubricants inside the gearbox, ensuring that all moving parts are properly lubricated. They are also used for preventing contamination of dust, dirt, water and other contaminants, and ensuring efficient operation [4].

For our gearbox we will use the Labyrinth sealing system. A labyrinth seal is a step up in design from the standard single seal or single seal with a dust lip. They feature a series of small, narrow channels to provide very high levels of resistance to flow. Labyrinth seals come in many variations, but they all feature a series of small, narrow channels to reduce the amount of dirt and debris in contact with the sealing surface. There are also non-contact labyrinth seals, used in high-speed applications to avoid the heat generation that accompanies contact seals, but they aren't a true seal and provide no sealing ability statically.

Retaining rings

Retaining rings, commonly known as snap rings, are a cost-efficient and reliable way to axially position and secure components like hubs and bearings onto shafts. Available in multiple varieties to meet different requirements, these rings fit into grooves on a shaft to hold other components in place. For our gearbox, we opted for simple snap rings. While they may not be the

top choice in every scenario, the advantages of using snap rings, including cost savings and precision, outweigh their potential drawbacks.

Maintenance

Regular maintenance is essential for the durability and dependability of industrial gearboxes. By performing routine tasks such as replacing gaskets, changing oil, and retightening bolts, we can significantly reduce the risk of breakdowns and operational interruptions, especially in industries with continuous machinery use. Proper maintenance not only ensures optimal performance but also lowers the costs associated with unexpected failures. Investing in gearbox upkeep not only maintains equipment integrity but also supports consistent operations and enhances productivity in industrial environments.

Ideally, maintenance should be conducted every six months. During this time, the lubrication system should be checked for contamination, oil levels should be monitored, and oil should be replaced as needed. Regular inspections of gaskets and seals are necessary to detect any leakage, damage, or wear.

Within the gearbox, it is important to examine the gear teeth for signs of wear, chipping, or cracking. The gear teeth contact patterns should be inspected to ensure proper alignment and meshing.

Given the vibration that gearboxes endure, bolts should be regularly checked for fatigue and damage. Retightening or replacing bolts, along with using appropriate thread lockers, is crucial to keeping them secure. Shaft alignment should also be verified during maintenance, as misalignment can lead to increased bearing wear and potential gearbox failure.

CAD Design (Ihab)

Drawings:

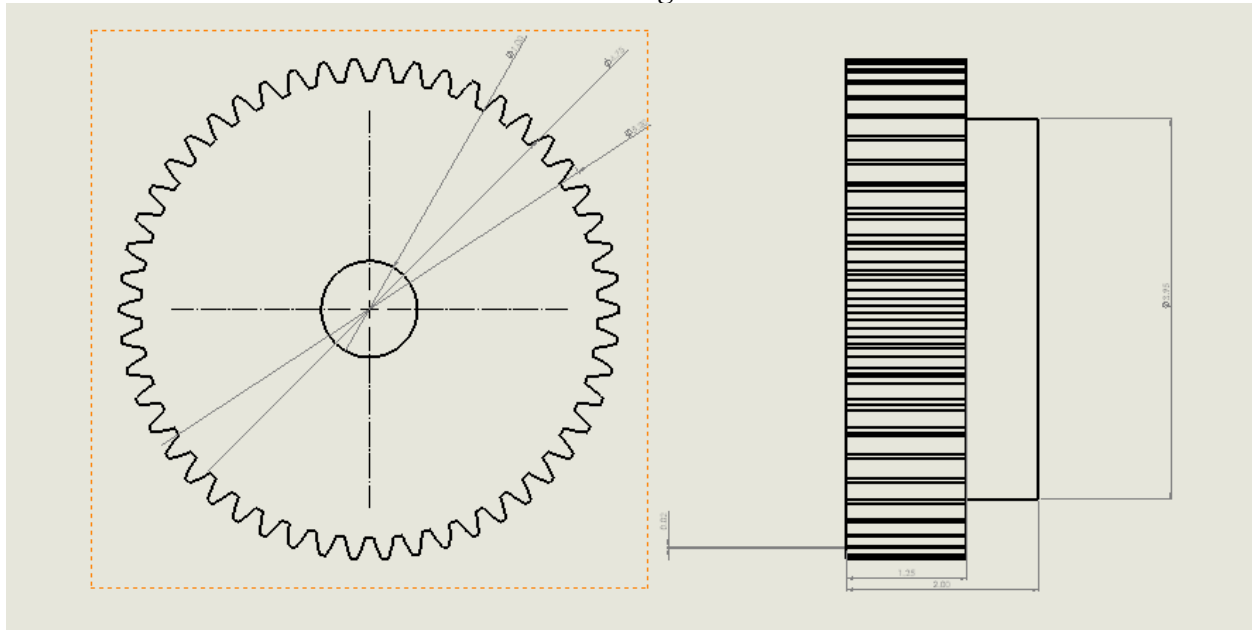


Figure 23: Straight spur gear

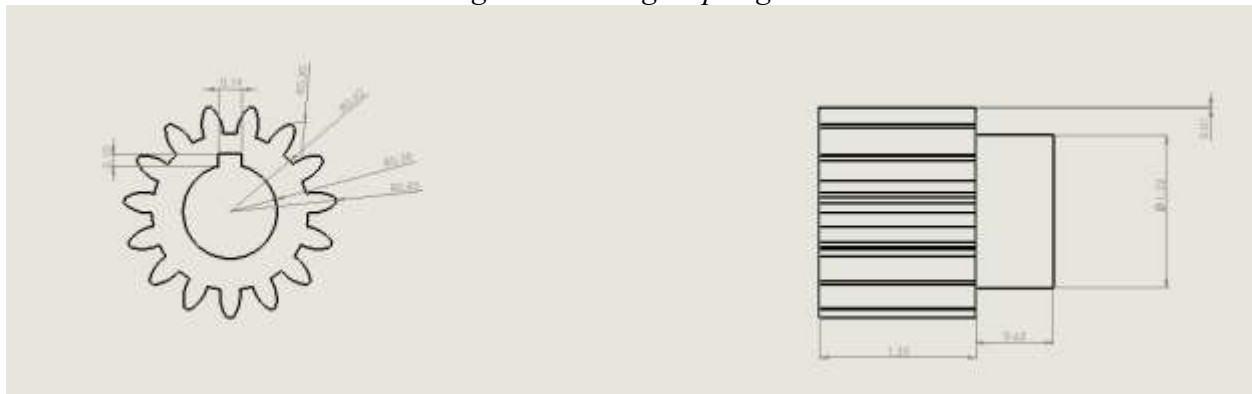


Figure 24: straight spur gear pinion

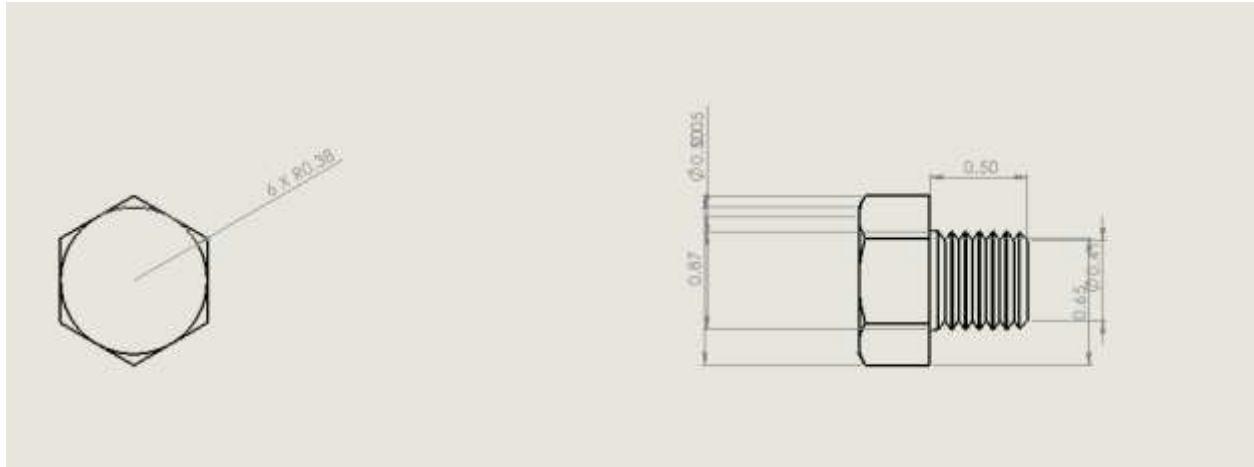


Figure 25: Bolt for input-output lubrication

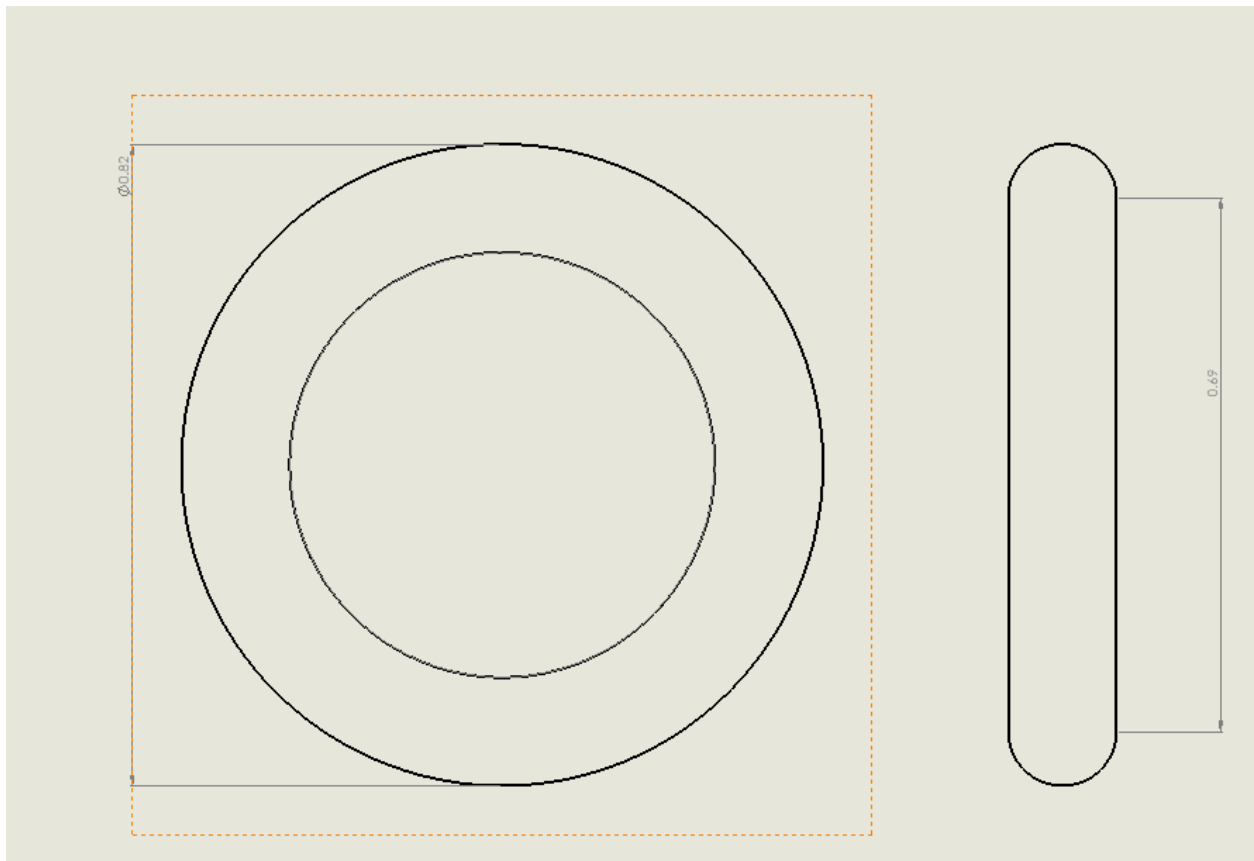


Figure 26: O-ring for sealing

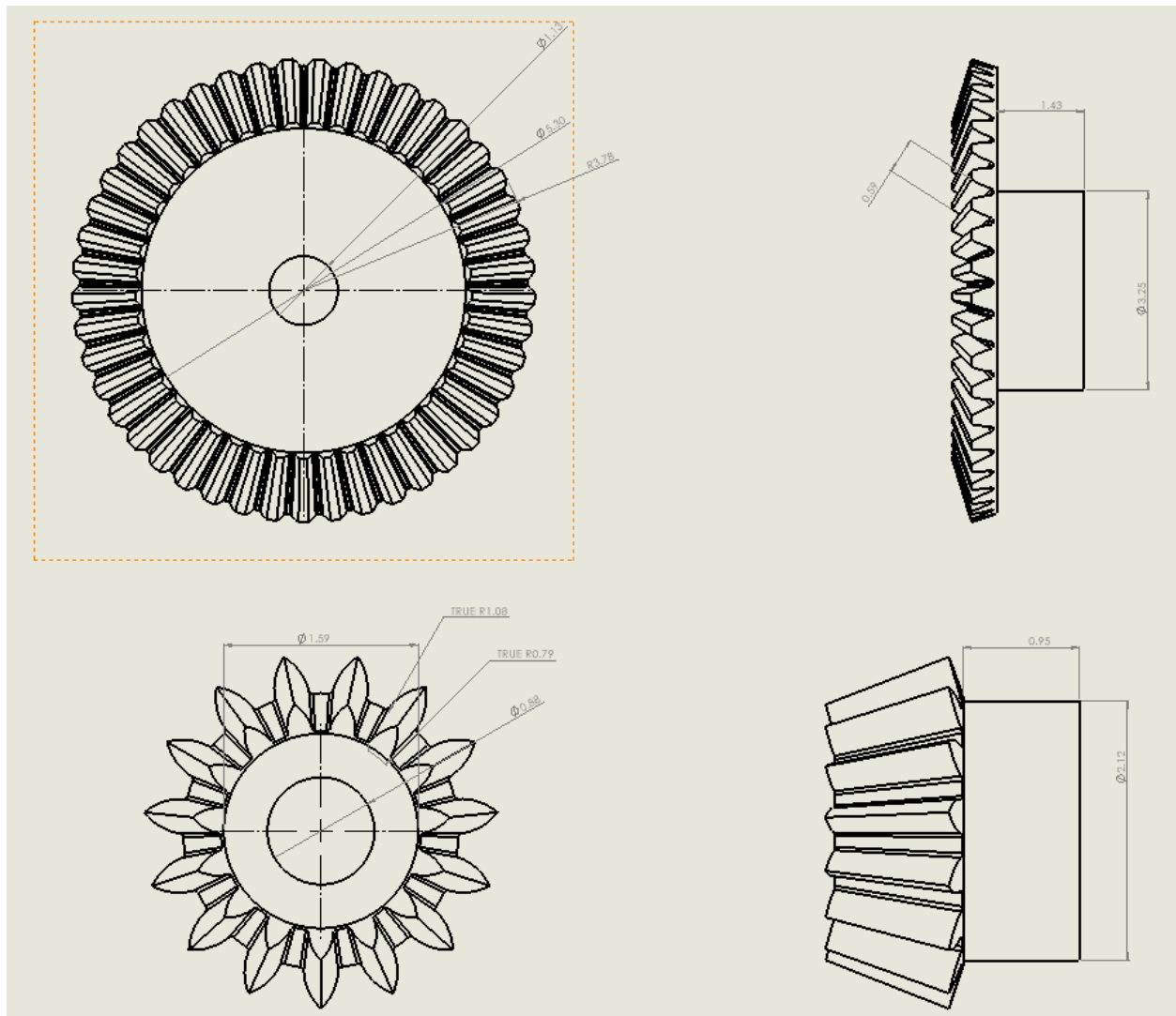


Figure 27: Pinion and Gears straight bevel

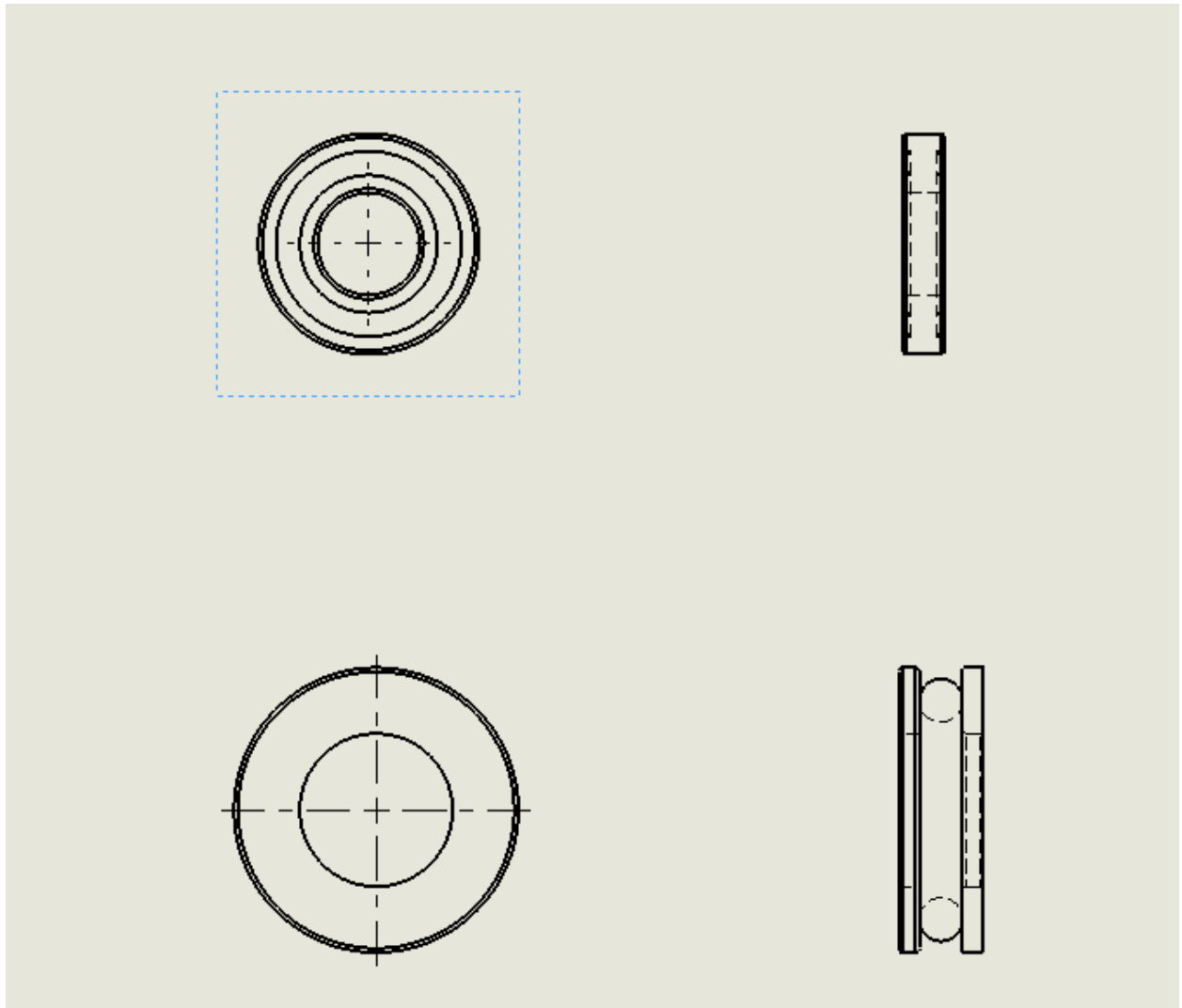


Figure 28: Deep groove ball bearing and thrust ball bearing

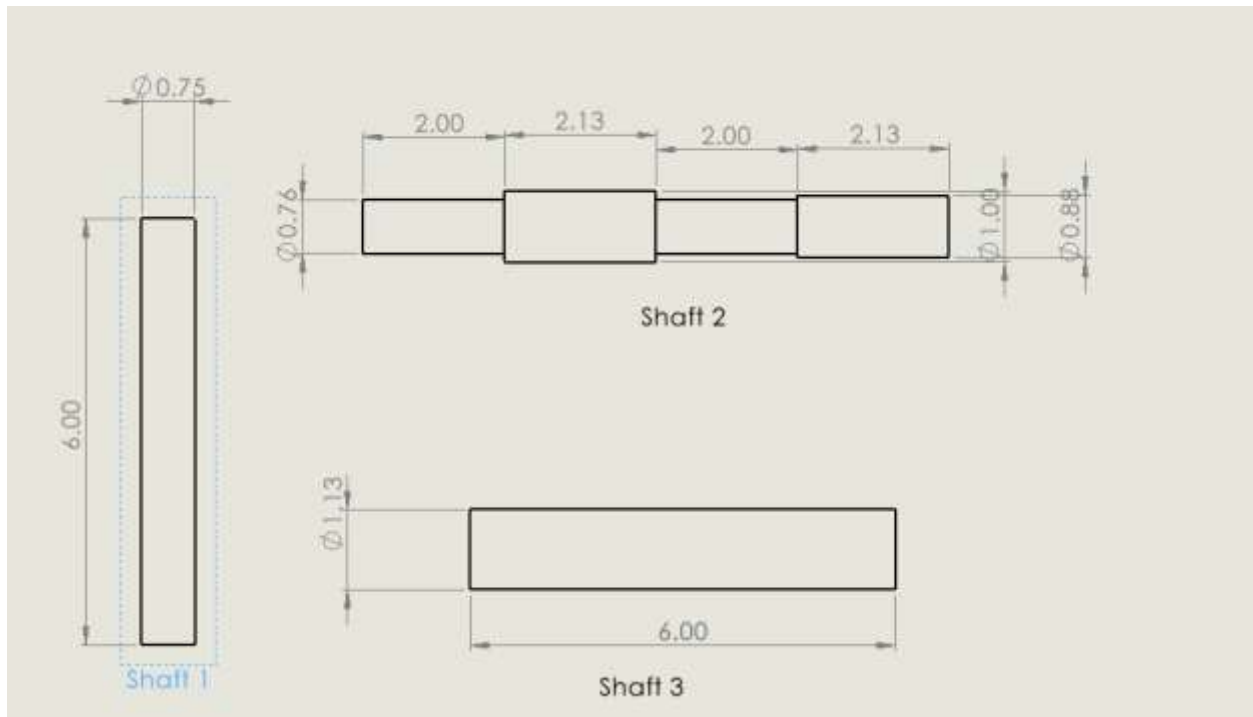


Figure 29: Shafts

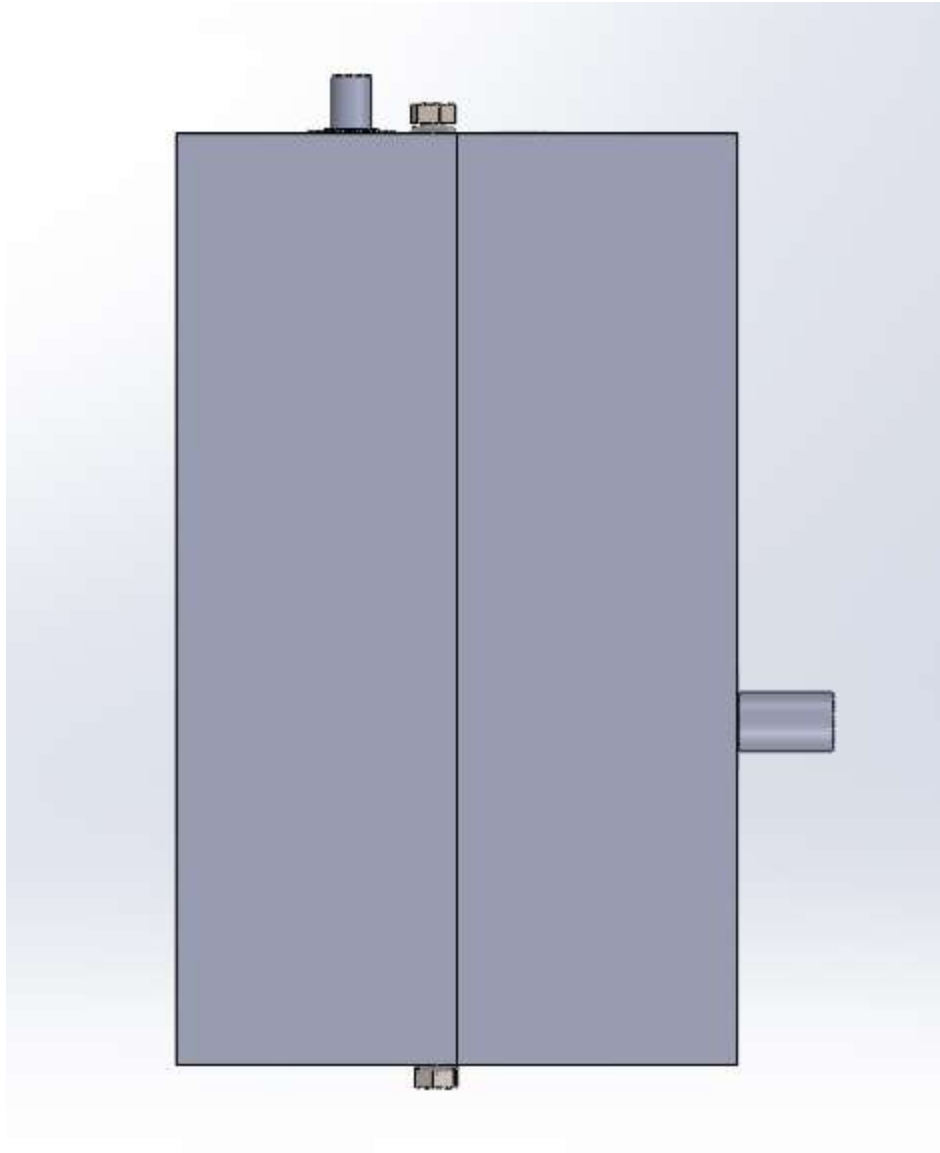


Figure 30: Front view gearbox

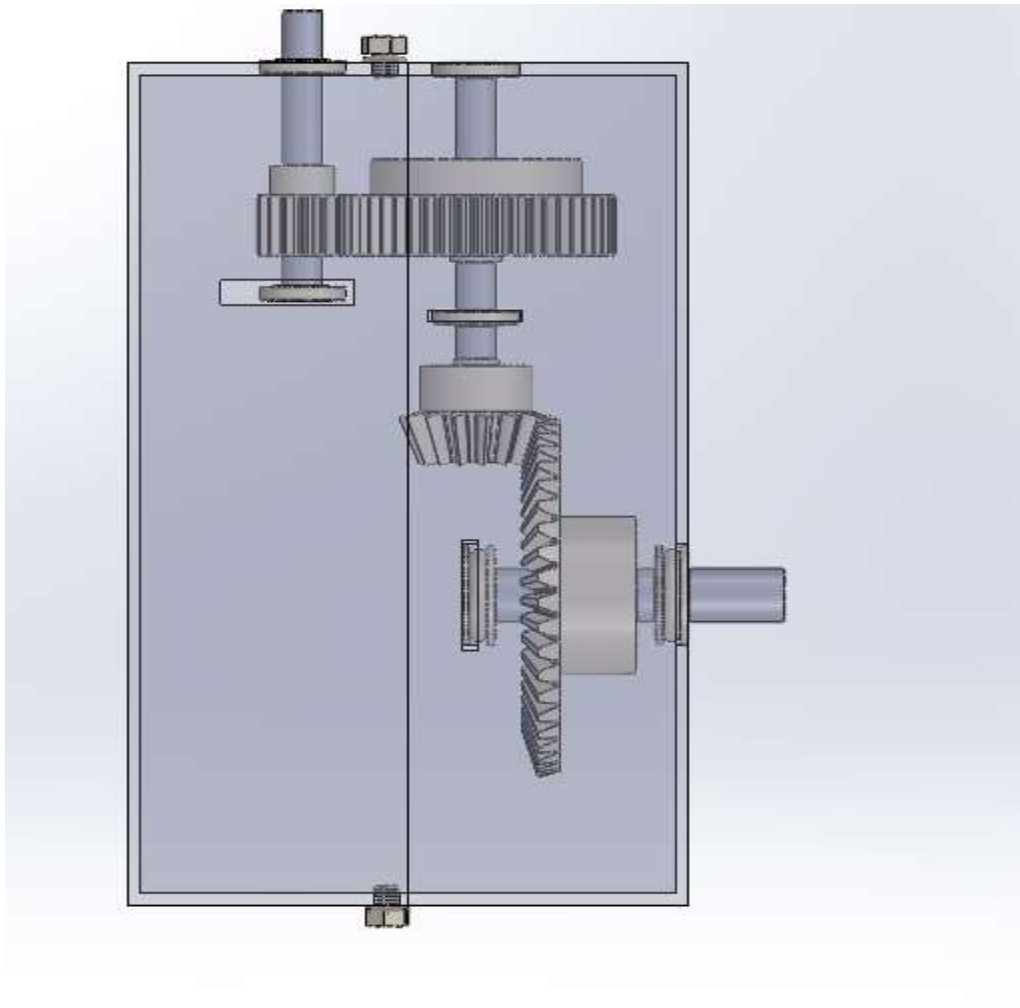
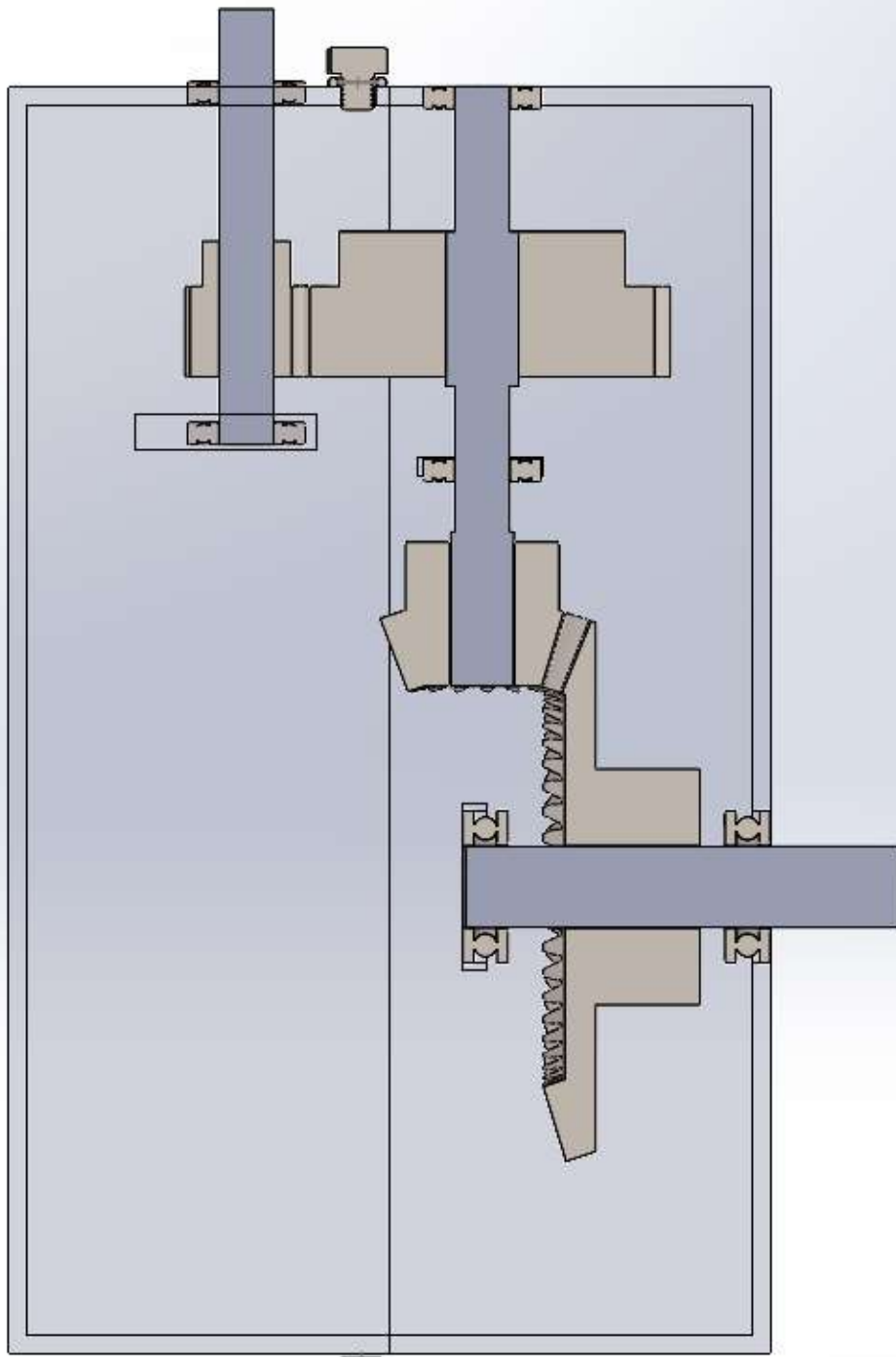


Figure 31: Front see through view of the gearbox

Figure 32: Front section view



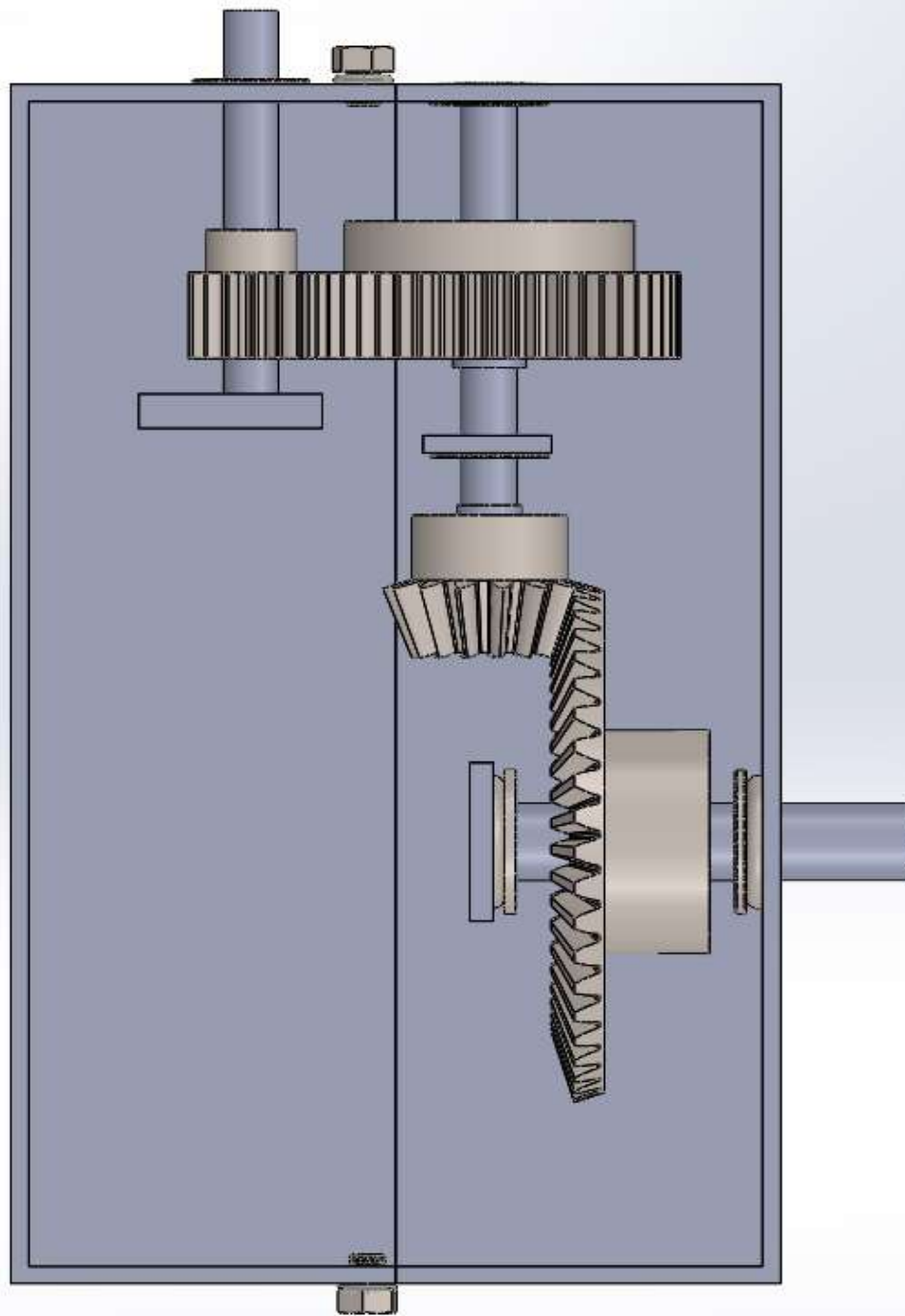


Figure 33:Front Section view of mechanism

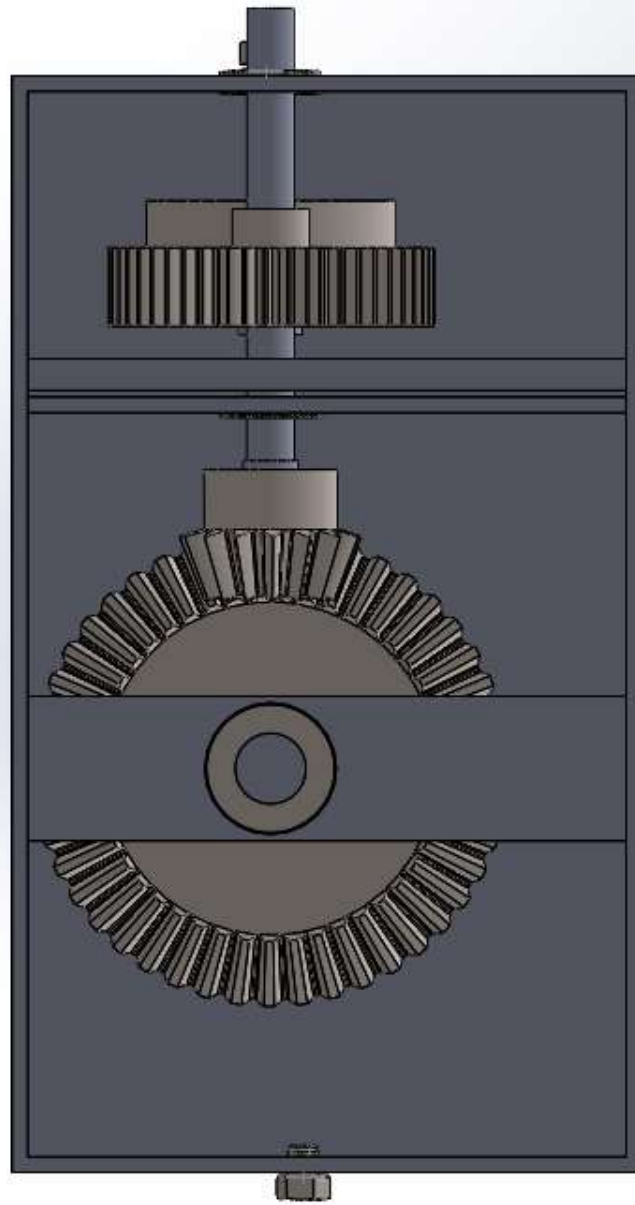


Figure 34: Right section view

Conclusion

In conclusion, the primary objective of this project was to design a stand-alone gear reducer with a speed reduction ratio of 10:1, suitable for applications such as conveyor belt systems. Throughout the design process, we explored the essential criteria required for manufacturing a robust gear reducer, including gear selection, stress calculations, shaft and bearing analysis, and key design considerations. Rigorous analysis ensured that our gear reducer met the necessary security factors, demonstrating its reliability and durability across various operational environments.

The project emphasized the importance of maintaining precise fits and tolerances, as well as the critical role of proper lubrication and maintenance procedures, including regular inspections of seals and retaining rings. These practices are crucial in ensuring the continued performance and longevity of the gear reducer.

Ultimately, this project highlights the practical application of theoretical mechanical engineering principles in the design and development of gear technology. The insights gained offer valuable experience in conceptualizing and executing complex engineering projects, underscoring the significance of combining technical knowledge with real-world applications.

References

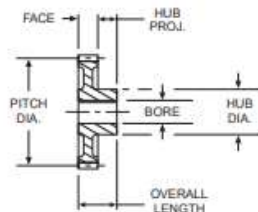
- [1] Boston Gear, “Gears, Couplings and Shaft Accessories P-1482-BG 50116 8/04.” [Online]. Available: www.bostongear.com
- [2] K. M. Robert C., Juvinall Marshek, *Fundamentals of machine component design*, 4th ed. Wiley.
- [3] “Synthetic Gear Oil Selection Guide.” Accessed: Jul. 20, 2024. [Online]. Available: <https://www.machinerylubrication.com/Read/167/synthetic-gear-oil>
- [4] “Gearbox Sealing Solutions: How A Little Component Does A Big Job.” Accessed: Jul. 20, 2024. [Online]. Available: <https://www.regalrexnord.com/regal-rexnord-insights/gearbox-sealing-solutions>

Appendix

SPUR GEARS

12 AND 10 DIAMETRAL PITCH CAST IRON AND STEEL

20° PRESSURE ANGLE
(Will not operate with 14½° spurs)



STANDARD TOLERANCES

DIMENSION	TOLERANCE
BORE All	±.0005
HUB DIA. All	±1/32



12 D.P.



10 D.P.

REFERENCE PAGES

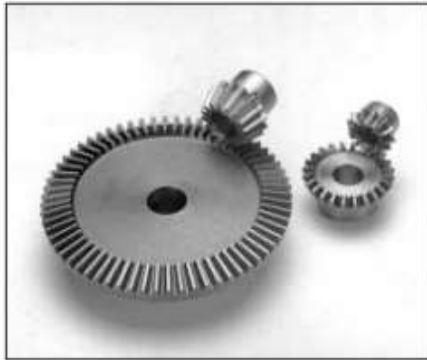
Alterations — 149
Horsepower Ratings — 47, 48
Lubrication — 149
Materials — 150

ALL DIMENSIONS IN INCHES
ORDER BY CATALOG NUMBER OR ITEM CODE

No. of Teeth	Pitch Dia.	Bore	Hub		Style See Page 150	Without Keyway or Setscrew		With Keyway and Setscrew†	
			Dia.	Proj.		Catalog Number	Item Code	Catalog Number	Item Code
12 DIAMETRAL PITCH						Face = 1.000" Outside Dia. = Pitch Dia. + .166" Overall Length = 1.000" + Hub Proj.			
CAST IRON									
60	5.000	.875	2.12	.88	B	YD60	10604	—	—
66	5.500					YD66	10606	—	—
72	6.000				YD72	10608	—	—	
84	7.000				YD84	10610	—	—	
96	8.000				YD96	10612	—	—	
108	9.000	1.000	2.25	.88	D	YD108	10614	—	—
120	10.000		2.25			YD120	10616	—	—
132	11.000		2.50		1.00	YD132	10618	—	—
144	12.000					YD144	10620	—	—
168	14.000					YD168	10622	—	—
192	16.000	YD192		10624		—	—		
216	18.000	2.75		YD216		10626	—	—	
10 DIAMETRAL PITCH						Face = 1.250" Outside Dia. = Pitch Dia. + .200" Overall Length = 1.250" + Hub Proj.			
STEEL									
12	1.200	.625	.92	.62	A	YF12	09962	YF12-5/8	46173
14	1.400		1.12			YF14	09964	YF14-5/8	46174
15	1.500		1.22			YF15	09966	YF15-3/4	46175
16	1.600	1.32	YF16	09968		YF16-3/4	46176		
18	1.800	.750	1.42	.62		YF18	09970	YF18-3/4	46177
20	2.000	.875	1.62	.62		—	—	YF18-7/8	46178
		1.000				YF20	09972	YF20-7/8	46179
24	2.400	.875	2.02	.62		—	—	YF20-1	46180
		1.000				YF24	09974	YF24-7/8	46181
25	2.500	.875	2.12	.62		—	—	YF24-1	46182
		1.000			YF25	09976	YF25-7/8	46183	
28	2.800	.875	2.42	.62	—	—	YF25-1	46184	
		1.000			YF28	09978	YF28-7/8	46185	
30	3.000	.875	2.00	.88	—	—	YF28-1	46186	
35	3.500		2.50		YF30A	10630	—	—	
40	4.000		2.95		YF35A	10632	—	—	
45	4.500	1.000	3.45	.88	YF40A	10634	—	—	
48	4.800		3.75		YF45A	10636	—	—	
50	5.000		3.95		YF48A	10638	—	—	
					YF50A	10640	—	—	
CAST IRON									
55	5.500	1.000	2.50	1.00	B	YF55	10642	—	—
60	6.000					YF60	10644	—	—
70	7.000				YF70	10646	—	—	
80	8.000				YF80	10648	—	—	
90	9.000				YF90	10650	—	—	
100	10.000	1.125	3.00	1.12	D	YF100	10652	—	—
120	12.000					YF120	10656	—	—
140	14.000				YF140	10658	—	—	
160	16.000				YF160	10660	—	—	
200	20.000				3.25	1.25	YF200B	10664	—

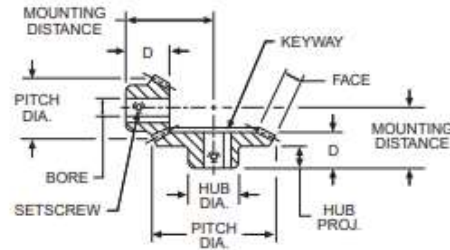
†All gears have standard keyway at 90° to setscrew. See Page 150.

STEEL, STAINLESS STEEL AND BRASS AND CAST IRON



All gears have "Coniflex"® tooth form.

All Hardened steel gears have teeth only hardened and are equipped with standard keyways and setscrews.



REFERENCE PAGES

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STANDARD TOLERANCES

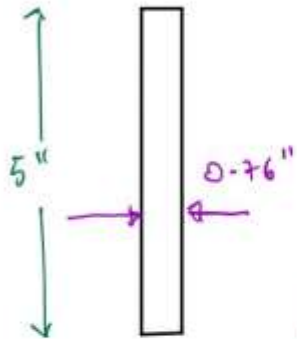
DIMENSION		TOLERANCE
BORE	All	±.0005

ALL DIMENSIONS IN INCHES
ORDER BY CATALOG NUMBER OR ITEM CODE

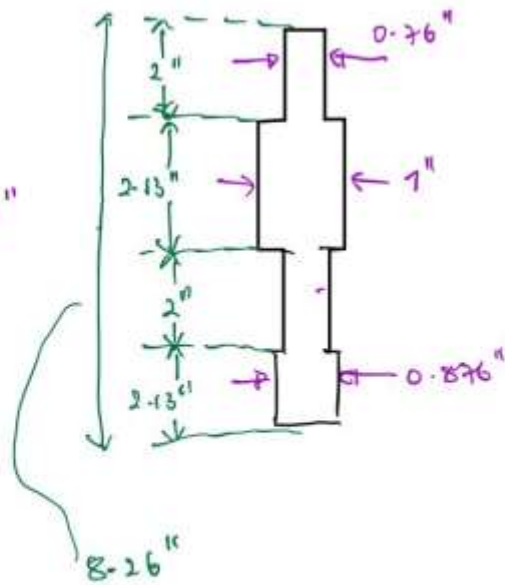
Ratio	No. of Teeth	Pitch Dia.	Face	Bore	MD *	D	Hub Dia.	Hub Proj.	Catalog Number	Item Code	Catalog Number	Item Code	Catalog Number	Item Code
8									STEEL UNHARDENED		STEEL HARDENED		CAST IRON GEARS	
DIAMETRAL PITCH													STEEL PINIONS	
2:1	40	5.000	.83	1.000	2.875	1.850	3.00	1.25	L156Y-G	12252	—	—	—	—
	20	2.500	.83	1.000	4.000	2.290	2.12	1.41	—	—	HL156Y-G	11886	—	—
	40	5.000	.83	1.000	2.875	1.850	3.00	1.25	—	—	—	—	PA528Y-G	12424
	20	2.500	.83	.875	4.000	2.290	2.12	1.41	—	—	—	—	PA528Y-P	12426
3:1	48	6.000	.84	.875	2.375	1.632	2.75	1.00	—	—	—	—	PA638Y-G	12436
	16	2.000	.84	.750	4.250	2.085	1.75	1.19	—	—	—	—	PA638Y-P	12438
	64	8.000	.85	1.000	2.750	1.882	2.75	1.25	—	—	—	—	PA848Y-G	12452
	16	2.000	.85	.875	5.250	2.105	1.88	1.22	—	—	—	—	PA848Y-P	12454
4:1	72	9.000	1.23	1.125	3.250	2.320	3.00	1.69	—	—	—	—	PA948Y-G	12460
	18	2.250	1.23	.875	5.750	2.470	2.13	1.22	—	—	—	—	PA948Y-P	12462
6														
DIAMETRAL PITCH														
2:1	36	6.000	1.07	1.125	3.500	2.260	3.25	1.50	L158Y-G	12278	—	—	—	—
	18	3.000	1.06	1.125	4.750	2.765	2.50	1.59	L158Y-P	12280	—	—	—	—
	36	6.000	1.07	1.750	3.500	2.260	3.25	1.50	—	—	HL158Y-G	11890	—	—
	18	3.000	1.06	1.125	4.750	2.765	2.50	1.59	—	—	HL158Y-P	11892	—	—
	36	6.000	1.07	1.125	3.500	2.260	3.25	1.50	—	—	—	—	PA626Y-G	12432
	18	3.000	1.07	1.000	4.750	2.765	2.50	1.59	—	—	—	—	PA626Y-P	12434
	42	7.000	1.06	1.125	3.750	2.305	3.50	1.50	—	—	—	—	PA726Y-G	12440
	21	3.500	1.06	1.000	5.000	2.515	2.50	1.25	—	—	—	—	PA726Y-P	12442
3:1	48	8.000	1.18	1.125	3.438	1.898	3.25	1.00	—	—	—	—	PA826Y-G	12448
	24	4.000	1.18	1.000	5.438	2.560	2.62	1.25	—	—	—	—	PA826Y-P	12450
	45	7.500	1.08	1.125	3.000	2.132	3.25	1.25	—	—	—	—	PA7536Y-G	12520
	15	2.500	1.08	.875	5.250	2.575	2.12	1.44	—	—	—	—	PA7536Y-P	12522
5														
DIAMETRAL PITCH														
2:1	30	6.000	1.05	1.125	3.500	2.257	3.25	1.38	—	—	—	—	PA625Y-G	12428
	15	3.000	1.05	1.000	4.375	2.390	2.62	1.28	—	—	—	—	PA625Y-P	12430
3:1	45	9.000	1.32	1.250	3.750	2.507	3.75	1.69	—	—	—	—	PA935Y-G	12456
	15	3.000	1.32	1.000	5.875	2.685	2.62	1.31	—	—	—	—	PA935Y-P	12458
4														
DIAMETRAL PITCH														
2:1	32	8.000	1.40	1.125	4.250	2.695	3.75	1.56	—	—	—	—	PA824Y-G	12444
	16	4.000	1.40	1.125	6.000	3.350	3.25	1.81	—	—	—	—	PA824Y-P	12446

*Mounting Distance (MD) must not be made less than dimension shown, see Page 144.

Input Shaft



Intermediate Shaft



Output Shaft

